

# Investigation

# 3

## Comparing and Scaling Rates

The following examples illustrate situations involving another strategy to compare numbers.

- My mom's car gets 45 miles per gallon on the expressway.
- We need two sandwiches for each person at the picnic.
- I earn \$3.50 per hour baby-sitting for my neighbor.
- The mystery meat label says 355 Calories per 6-ounce serving.
- My brother's top running rate is 8.5 kilometers per hour.

Each of these statements compares two different quantities. For example, one compares miles to gallons of gas. A comparison of two quantities measured in different units is a **rate**. You have used rates in earlier problems. For example, you used rates in finding pizza per person.

### Getting Ready for Problem 3.1

- What two quantities are being compared in the rate statements above?
- Which of the rate statements is different from the others?



## 3.1 Technology on Sale

Stores, catalogs, and Web sites often use rates in their ads. The ads sometimes give the costs for several items. You might see an offer like the one shown at the right.

### Calculators for School

Fraction: \$120 for 20  
Scientific: \$240 for 15  
Graphing: \$800 for 10



The listed prices are for orders of 10, 15, or 20 calculators. But it's possible to figure the price for any number you want to purchase. One way to figure those prices is to build a *rate table*. A rate table is started below.

Price of Calculators for Schools

Number Purchased	1	2	3	4	5	10	15	20
Fraction	■	■	■	■	■	■	■	\$120
Scientific	■	■	■	■	■	■	\$240	■
Graphing	■	■	■	■	■	\$800	■	■

### Problem 3.1 Making and Using a Rate Table

Suppose you take orders over the phone for the calculator company. You should be quick with price quotes for orders of different sizes.

- A.** Build a rate table like the one above. Fill in prices for each type of calculator for orders of the sizes shown.

Use your rate table to answer Questions B–F.

- B.** How much does it cost to buy 53 fraction calculators? How much to buy 27 scientific calculators? How much to buy 9 graphing calculators?
- C.** How many fraction calculators can a school buy if it can spend \$390? What if the school can spend only \$84?
- D.** How many graphing calculators can a school buy if it can spend \$2,500? What if the school can spend only \$560?
- E.** What *arithmetic operation* (addition, subtraction, multiplication or division) do you use to find the cost per calculator?
- F.** Write an equation for each kind of calculator to show how to find the price for any number ordered.

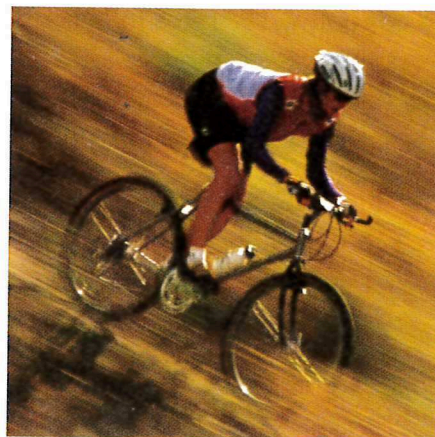
**ACE** Homework starts on page 40.



## 3.2 Time, Rate, and Distance

Sascha cycled on a route with different kinds of conditions. Sometimes he went uphill, sometimes he went mostly downhill. Sometimes he was on flat ground. He stopped three times to record his time and distance:

- Stop 1: 5 miles in 20 minutes
- Stop 2: 8 miles in 24 minutes
- Stop 3: 15 miles in 40 minutes



### Problem 3.2 Finding Rates

Show your work. Label any rate that you find with appropriate units.

- Find Sascha's rate in miles per hour for each part of the route.
- On which part was Sascha cycling fastest? On which part was he cycling slowest?
  - How do your calculations in Question A support your answers?
- Suppose you can maintain a steady rate of 13 miles per hour on a bike. How long will it take you to travel the same distance Sascha traveled in 1 hour and 24 minutes?
- Suppose you were racing Sascha. What steady rate would you have to maintain to tie him?

**ACE** Homework starts on page 40.

### Did You Know?

The highest rate ever recorded on a pedal-powered bicycle was 166.944 miles per hour. Fred Rempelberg performed this amazing feat on October 3, 1995, at the Bonneville Salt Flats in Utah. He was able to reach this rate by following a vehicle. The vehicle acted as a windshield for him and his bicycle.



For: Information about speed records  
Web Code: ane-9031

### 3.3 Comparing CD Prices

The ads below use rates to describe sale prices. To compare prices in sales such as these, it's often useful to find a unit rate. A **unit rate** is a rate in which one of the numbers being compared is 1 unit. The comparisons "45 miles per gallon," "\$3.50 per hour," "8.5 kilometers per hour," and "two sandwiches for each person" are all unit rates. "Per gallon" means "for one gallon" and "per hour" means "for one hour."



#### Problem 3.3 Unit Rates and Equations

Use unit rates to compare the ad prices and to find the costs of various numbers of CDs at each store.

- Which store has the lower price per CD?
- For each store, write an equation (a rule) that you can use to calculate the cost  $c$  for any purchase of  $n$  compact discs.
- Use the equations you just wrote for Question B. Write new equations to include 5% sales tax on any purchase.



- D. Suppose a Web site sells CDs for \$8.99 per disc. There is no tax, but there is a shipping charge of \$5 for any order. Write an equation to give the cost  $c$  of any order for  $n$  discs from the Web site.
- E. Use your equations from Question C or make a rate table to answer each question.
1. How many discs do you have to order from the Web site to get a better deal than buying from Music City?
  2. How many discs do you have to order from the Web site to get a better deal than buying from CD World?

**ACE** Homework starts on page 40.

## 3.4 What Does Dividing Tell You?

In this problem, the questions will help you decide which way to divide when you are finding a unit rate. The questions will also help you with the meaning of the quotient after you divide.

### Getting Ready for Problem 3.4

Dario has two options for buying boxes of pasta. At CornerMarket he can buy seven boxes of pasta for \$6. At SuperFoodz he can buy six boxes of pasta for \$5.

At CornerMarket, he divided 7 by 6 and got 1.1666667. He then divided 6 by 7 and got 0.85714286. He was confused. What do these numbers tell about the price of boxes of pasta at CornerMarket?

Decide which makes more sense to you. Use that division strategy to compare the two store prices. Which store offers the better deal?

### Problem 3.4 Two Different Rates

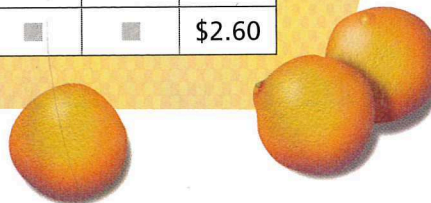
Use division to find unit rates to solve the following questions. Label each unit rate.

**A.** SuperFoodz has oranges on sale at 10 for \$2.

1. What is the cost per orange?
2. How many oranges can you buy for \$1?
3. What division did you perform in each case? How did you decide what each division means?
4. Complete this rate table to show what you know.

**Cost of Oranges at SuperFoodz**

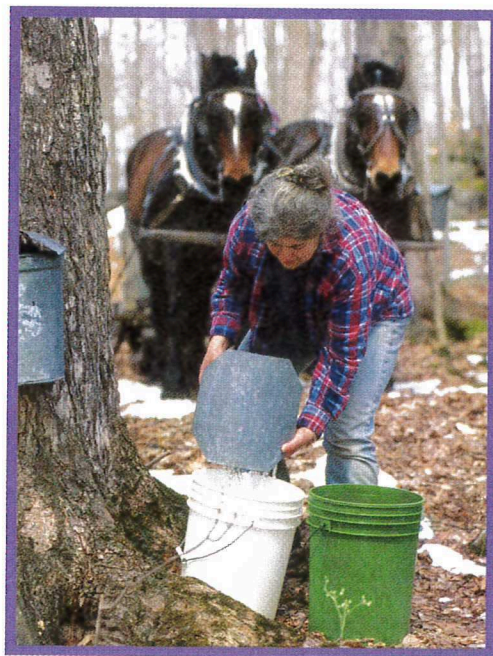
Oranges	10	■	1	20	11	■
Cost	\$2.00	\$1.00	■	■	■	\$2.60



**B.** Noralie used 22 gallons of gas to go 682 miles.

1. What are the two unit rates that she might compute?
2. Compute each unit rate and tell what it means.
3. Which seems more useful to you? Why?

- C. It takes 100 maple trees to make 25 gallons of maple syrup.
1. How many maple trees does it take for 1 gallon of syrup?
  2. How much syrup can you get from one maple tree?



- D. A 5-minute shower requires about 18 gallons of water.
1. How much water per minute does a shower take?
  2. How long does a shower last if you use only 1 gallon of water?
- E. 1. At the CornerMarket grocery store, you can buy eight cans of tomatoes for \$9. The cans are the same size as those at CannedStuff, which sells six cans for \$5. Are the tomatoes at CornerMarket a better buy than the tomatoes at CannedStuff?
2. What comparison strategies did you use to choose between CornerMarket and CannedStuff tomatoes? Why?

**ACE** Homework starts on page 40.



## Applications

The problems that follow will give you practice in using rates (especially unit rates) in different situations. Be careful to use measurement units that match correctly in the rates you compute.

1. Maralah can drive her car 580 miles at a steady speed using 20 gallons of gasoline. Make a rate table showing the number of miles her car can be driven at this speed. Show 1, 2, 3, ..., and 10 gallons of gas.
2. Joel can drive his car 450 miles at a steady speed using 15 gallons of gasoline. Make a rate table showing the number of miles his car can be driven at this speed. Show 1, 2, 3, ..., and 10 gallons of gas.
3. Franky's Trail Mix Factory gives customers the following information. Use the pattern in the table to answer the questions.

**Caloric Content of  
Franky's Trail Mix**

Grams of Trail Mix	Calories
50	150
150	450
300	900
500	1,500

- a. Fiona eats 75 grams of trail mix. How many Calories does she eat?
- b. Rico eats trail mix containing 1,000 Calories. How many grams of trail mix does he eat?
- c. Write an equation that you can use to find the number of Calories in any number of grams of trail mix.
- d. Write an equation that you can use to find the number of grams of trail mix that will provide any given number of Calories.



For Exercises 4–8, you will explore relationships among time, rate, and distance.

4. When she drives to work, Louise travels 10 miles in about 15 minutes. Kareem travels 23 miles in about 30 minutes. Who has the faster average speed?
5. Rolanda and Mali ride bikes at a steady pace. Rolanda rides 8 miles in 32 minutes. Mali rides 2 miles in 10 minutes. Who rides faster?
6. Fasiz and Dale drive at the same speed along a road. Fasiz drives 8 kilometers in 24 minutes. How far does Dale drive in 6 minutes?
7. On a long dirt road leading to camp, buses travel only 6 miles in 10 minutes.
  - a. At this speed, how long does it take the buses to travel 18 miles?
  - b. At this speed, how far do the buses go in 15 minutes?
8. **Multiple Choice** Choose the fastest walker.
  - A. Montel walks 3 miles in 1 hour.
  - B. Jerry walks 6 miles in 2 hours.
  - C. Phil walks 6 miles in 1.5 hours.
  - D. Rosie walks 9 miles in 2 hours.
9. The dairy store says it takes 50 pounds of milk to make 5 pounds of cheddar cheese.
  - a. Make a rate table showing the amount of milk needed to make 5, 10, 15, 20, ..., and 50 pounds of cheddar cheese.
  - b. Make a coordinate graph showing the relationship between pounds of milk and pounds of cheddar cheese. First, decide which variable should go on each axis.
  - c. Write an equation relating pounds of milk  $m$  to pounds of cheddar cheese  $c$ .
  - d. Explain one advantage of each method (the graph, the table, and the equation) to express the relationship between milk and cheddar cheese production.



- 10.** A dairy manager says it takes 70 pounds of milk to make 10 pounds of cottage cheese.
- Make a rate table for the amount of milk needed to make 10, 20, . . . , and 100 pounds of cottage cheese.
  - Make a graph showing the relationship between pounds of milk and pounds of cottage cheese. First, decide which variable should go on each axis.
  - Write an equation relating pounds of milk  $m$  to pounds of cottage cheese  $c$ .
  - Compare the graph in part (b) to the graph in Exercise 9. Explain how they are alike and how they are different. What is the cause of the differences between the two graphs?
- 11.** A store sells videotapes at \$3.00 for a set of two tapes. You have \$20. You can split a set and buy just one tape for the same price per tape as the set.
- How many tapes can you buy?
  - Suppose there is a 7% sales tax on the tapes. How many can you buy? Justify your solution.
- 12.** Study the data in these rate situations. Then write the key relationship in three ways:
- in fraction form with a label for each part
  - as two different unit rates with a label for each rate
- Latanya's 15-mile commute to work each day takes an average of 40 minutes.
  - In a 5-minute test, one computer printer produced 90 pages of output.
  - An advertisement for a Caribbean cruise trip promises 168 hours of fun for only \$1,344.
  - A long-distance telephone call lasts 20 minutes and costs \$4.50.



For: Help with Exercise 11  
Web Code: ane-3311

## Connections

Rewrite each equation, replacing the variable with a number that makes a true statement.

13.  $\frac{4}{9} \times n = 1\frac{1}{3}$

14.  $n \times 2.25 = 90$

15.  $n \div 15 = 120$

16.  $180 \div n = 15$

17. Write two fractions with a product between 10 and 11.

18. Write two decimals with a product between 1 and 2.

**A recent world-champion milk producer was a 4-year-old cow from Marathon, Wisconsin. The cow, Muranda Oscar Lucinda, produced a record 67,914 pounds of milk in one year! Use this information for Exercises 19–22.**

19. Look back at your answers to Exercise 10. How much cottage cheese could be made from the amount of milk that Muranda Oscar Lucinda produced during her record year?
20. The average weight of a dairy cow is 1,500 pounds. How many dairy cows would be needed to equal the weight of the cottage cheese you found in Exercise 19?
21. One gallon of milk weighs about 8.7 pounds. Suppose a typical milk bucket holds about 3 gallons. About how many milk buckets would Muranda Oscar Lucinda's average daily production of milk fill?
22. One pound of milk fills about two glasses. About how many glasses of milk could you fill with Muranda Oscar Lucinda's average daily production of milk?
23. Some campers bike 10 miles for a nature study. Use this setting to write questions that can be answered by solving each equation. Find the answers, and explain what they tell about the bike ride.
- a.  $10 \div 8 = \blacksquare$       b.  $1.2 \times \blacksquare = 10$       c.  $\blacksquare \div 2 = 5$

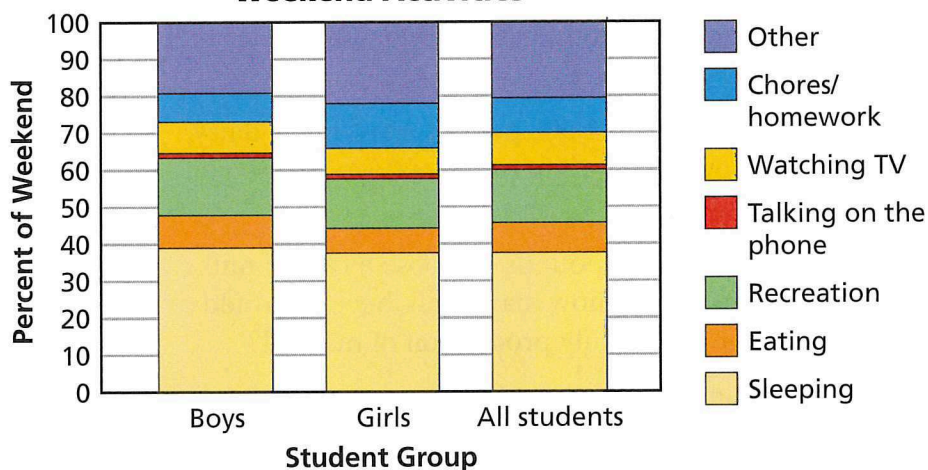


The table shows the mean times that students in one seventh-grade class spend on several activities during a weekend. The data are also displayed in the stacked bar graph below the table. Use both the table and the graph for Exercises 24 and 25.

**Weekend Activities (hours)**

Category	Boys	Girls	All Students
Sleeping	18.8	18.2	18.4
Eating	4.0	2.7	3.1
Recreation	7.8	6.9	7.2
Talking on the Phone	0.5	0.7	0.6
Watching TV	4.2	3.0	3.4
Doing Chores and Homework	3.6	5.8	5.1
Other	9.1	10.7	10.2

**Weekend Activities**



24. The stacked bar graph was made using the data from the table. Explain how it was constructed.
25. Suppose you are writing a report summarizing the class's data. You have space for either the table or the graph, but not both. What is one advantage of including the table? What is one advantage of including the stacked bar graph?

26. This table shows how to convert liters to quarts.

- About how many liters are in 5.5 quarts?
- About how many quarts are in 5.5 liters?
- Write an equation for a rule that relates liters  $L$  to quarts  $Q$ .

Liters	Quarts
1	1.06
4	4.24
5	5.30
9	9.54

Express each of the relationships in Exercises 27–31 as a unit rate. Label each unit rate with measurement units.

- 12 cents for 20 beads
- 8 cents for 10 nails
- 405 miles on 15 gallons of gasoline
- 3 cups of water for 2 cups of orange concentrate
- \$4 for 5 cans of soup

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 Web Code: ana-3354

- The two clocks shown below are geometrically similar. One is a reduction of the other. Each outside edge of the larger clock is 2 centimeters long. Each outside edge of the smaller clock is 1.6 centimeters long.



- Write an equation relating the length  $L$  of any part of the large clock to the length  $S$  of the corresponding part of the small clock.
- Write an equation relating the area  $R$  of any part of the large clock to the area  $M$  of the corresponding part of the small clock.
- Write a decimal scale factor relating lengths in the large clock to lengths in the small clock. Explain how that scale factor is like a unit rate.



## Extensions

33. Chemistry students analyzed the contents of rust. They found that it is made up of iron and oxygen. Tests on samples of rust gave these data.

**Contents of Rust**

Amount of Rust (g)	Amount of Iron (g)	Amount of Oxygen (g)
50	35.0	15.0
100	70.0	30.0
135	94.5	40.5
150	105.0	45.0

- Suppose the students analyze 400 grams of rust. How much iron and how much oxygen should they find?
- Is the ratio of iron to oxygen the same in each sample? If so, what is it? If not, explain.
- Is the ratio of iron to total rust the same in each sample? If so, what is it? If not, explain.



34. A cider mill owner has pressed 240 liters of apple juice. He has many sizes of containers in which to pack the juice.
- The owner wants to package all the juice in containers of the same size. Copy and complete this table to show the number of containers of each size needed to hold the juice.

**Containers Needed by Volume**

Volume of Container (liters)	10	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$
Number of Containers Needed	■	■	■	■	■	■	■

- Write an equation that relates the volume  $v$  of a container and the number  $n$  of containers needed to hold 240 liters of juice.



# Mathematical Reflections

## 3

**In this investigation, you learned to compare rates, to find unit rates, and to use rates to make tables and graphs and to write equations. The following questions will help you summarize what you have learned.**

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Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

The Picked Today fruit stand sells three green peppers for \$1.50.

1.
  - a. Describe the process for finding a unit rate for the peppers.
  - b. Find two different unit rates to express the relationship between peppers and price. Explain what each unit rate tells.
  - c. Fresh Veggie sells green peppers at five for \$2.25. Compare Picked Today pepper prices with Fresh Veggie prices using two different kinds of unit rates.
  - d. How do you decide whether the larger unit rate or the smaller unit rate is the better buy?
2. How would you construct a rate table for green pepper prices at the two vegetable stands? Explain what the entries in the table tell.
3.
  - a. How would you write an equation to show the price for  $n$  peppers bought at Picked Today?
  - b. Explain how the unit rate is used in writing the equation.

