Setting Up and Solving Proportions

There are many ways to solve problems such as the ones on the previous page. One standard way is to create two ratios to represent the information in the problem. Then set these two ratios equal to each other to form a proportion. A **proportion** is an equation that states two ratios are equal.

For example, in the problem about doctors, you have enough information to write one ratio. Then write a proportion to find the missing quantity. There are four different ways to write a proportion representing the data in the problem.

Write the known ratio of male to female doctors. Complete the proportion with the ratio of actual numbers of doctors.

$$\frac{15 \text{ (male)}}{4 \text{ (female)}} = \frac{450{,}000 \text{ males}}{x \text{ females}}$$

Write a ratio of male to male data. Complete the proportion with female to female data.

$$\frac{15 \text{ (male)}}{450{,}000 \text{ males}} = \frac{4 \text{ (female)}}{x \text{ females}}$$

Write the known ratio of female to male doctors. Complete the proportion with the ratio of actual numbers of doctors.

$$\frac{4 \text{ (female)}}{15 \text{ (male)}} = \frac{x \text{ females}}{450{,}000 \text{ males}}$$

Write a different ratio of male to male data. Complete the proportion with female to female data.

$$\frac{450{,}000 \text{ males}}{15 \text{ (male)}} = \frac{x \text{ females}}{4 \text{ (female)}}$$

Using your knowledge of equivalent ratios, you can now find the number of female doctors from any one of these proportions.

Does any arrangement seem easier than the others?

Analyze the "Similar Figures" problem in the introduction.

The scale factor relating two similar figures is 2. One side of the larger figure is 10 centimeters long. How long is the corresponding side of the smaller figure?

- The scale factor means that the lengths of the sides of the larger figure are 2 times the lengths of the sides of the smaller. What is the ratio of the side lengths of the smaller figure to those of the larger figure?

- Write a proportion to represent the information in the problem.

- Solve your proportion to find the length of the corresponding side of the smaller figure.

Problem 4.1 Setting Up and Solving Proportions

A. Figure out whether each student's thinking about each line in the following problem is correct. Explain.

Dogs outnumber cats in an area by a ratio of 9 to 8. There are 180 dogs in the area. How many cats are there?

Adrianna's Work:

$$\frac{9 \text{ dogs}}{8 \text{ cats}} = \frac{180 \text{ dogs}}{x \text{ cats}}$$

$$\frac{9}{8} \times \frac{20}{20} = \frac{180}{160}$$

$$\frac{180}{160} = \frac{180}{x}$$

$$x = 160$$

1. Why did Adrianna multiply by $\frac{20}{20}$? How did she find what to multiply by?

2. What does this proportion tell you about the denominators? Why?

3. Is the answer correct? Explain.

Joey's Work:

$$\frac{8 \text{ cats}}{9 \text{ dogs}} = \frac{x \text{ cats}}{180 \text{ dogs}}$$

$$\frac{8}{9} = \frac{80}{90} = \frac{160}{180}$$

There are 160 cats.

4. What strategy did Joey use?

5. Why can he make this claim?

B. 1. Calculators are on sale at a price of $1,000 for 20. How many can be purchased for $1,250? Write and solve a proportion that represents the problem. Explain.

2. Country music is the primary format of 20% of American radio stations. There are about 10,600 radio stations in the United States. About how many stations focus on country music?

C. Use the reasoning you applied in Question B to solve these proportions for the variable x. Explain.

1. $\dfrac{8}{5} = \dfrac{32}{x}$ **2.** $\dfrac{7}{12} = \dfrac{x}{9}$ **3.** $\dfrac{25}{x} = \dfrac{5}{7}$ **4.** $\dfrac{x}{3} = \dfrac{8}{9}$

D. Use proportions to find the missing lengths in the following similar shapes.
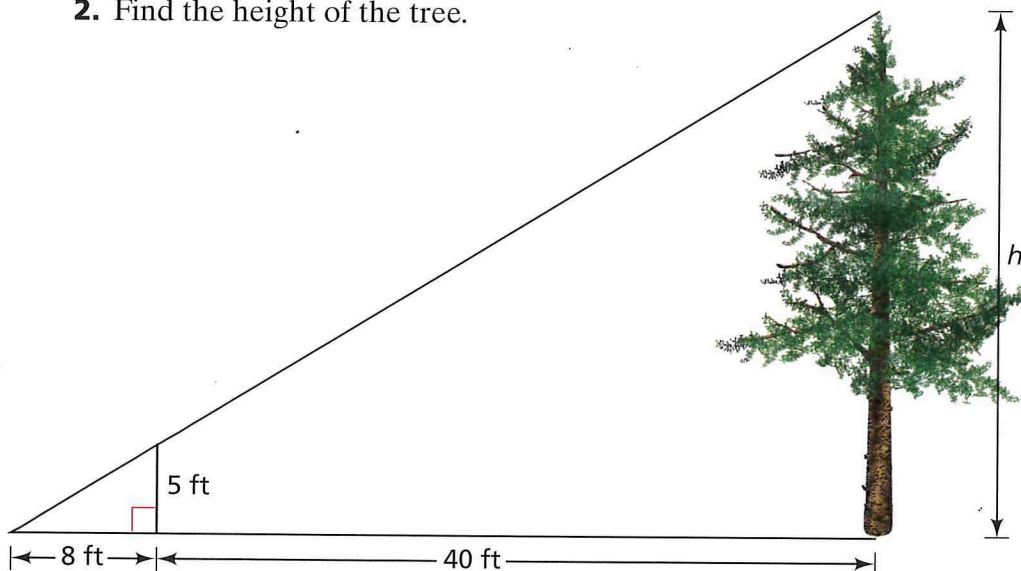
1.
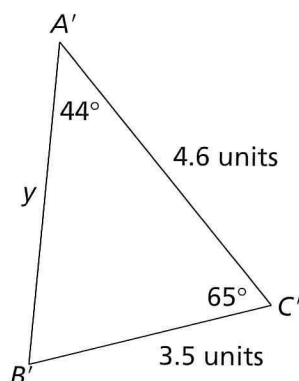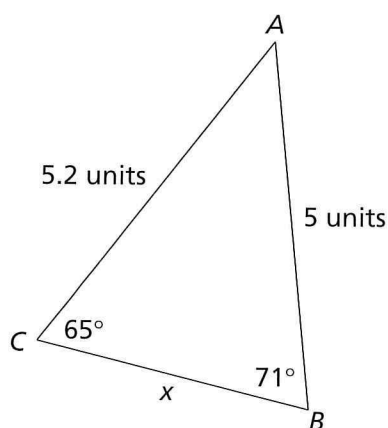
active math online

For: Scaling Figures Activity
Visit: PHSchool.com
Web Code: and-3401

2. Find the height of the tree.

ACE Homework starts on page 55.

4.2 Everyday Use of Proportions

In our everyday lives, we often need to solve proportion problems. So do bakers, tailors, designers, and people in many other occupations.

You may have heard someone say, "A pint is a pound the world around." This saying suggests how to compare liquid measures with weight. It tells us that a pint of liquid weighs about a pound. If you drink a quart of milk a day, you might ask,

"About how much does a quart of liquid weigh?"

Problem 4.2 Applications of Proportions

A. Jogging 5 miles burns about 500 Calories. How many miles will Tanisha need to jog to burn off the 1,200-Calorie lunch she ate?

B. Tanisha jogs about 8 miles in 2 hours. How long will it take her to jog 12 miles?

C. Sam's grandmother says that "a stitch in time saves nine."

   1. What do you think Sam's grandmother means?

   2. Sam's grandmother takes 25 stitches in time. How many does she save?

D. Imani gives vitamins to her adult dogs. The recommended dosage is 2 teaspoons per day for adult dogs weighing 20 pounds. She needs to give vitamins to Bruiser, who weighs 75 pounds, and to Dust Ball, who weighs 7 pounds. What is the correct dosage for each dog?

E. The scale factor relating two similar figures is 1.8. One side of the larger figure is 12 centimeters. How long is the corresponding side of the smaller figure?

ACE **Homework starts on page 55.**

4.3 Developing Strategies for Solving Proportions

When mathematicians find the same kind of problem occurring often, they look for a systematic method, or algorithm, that can be applied in each case.

So far in this investigation, you have found ways to solve proportions in specific cases with nice numbers. Now you will develop general strategies that will guide you in solving proportions when the numbers are not so nicely related.

Problem 4.3 Developing Strategies for Solving Proportions

A. A jet takes 10 miles to descend 4,000 feet. How many miles does it take for the jet to descend 5,280 feet?

  1. Set up two different proportions that can be solved to answer the question.

  2. Solve one of your proportions by whatever method you choose. Check to see that your answer makes sense.

B. Jack works at a restaurant and eats one enchilada for lunch every day that he works. He figures that he ate 240 enchiladas last year. Three enchiladas have a total of 705 Calories. How many Calories did he take in last year from eating enchiladas?

  1. Set up a proportion that can be solved to answer the question.

  2. Solve your proportion. Check to see that your answer makes sense.

  3. Describe each step in your solution strategy.

  4. Can your strategy be used to solve any proportion? Explain.

  5. How many Calories did he eat for lunch each working day?

C. In Pinecrest Middle School, there are 58 sixth-graders, 76 seventh-graders, and 38 eighth-graders. The school council is made up of 35 students who are chosen to represent all three grades fairly.

   1. Write fractions to represent the part of the school population that is in each grade.

   2. Use these fractions to write and solve proportions that will help you determine a fair number of students to represent each grade on the school council. Explain.

   3. How would the number of students from each grade change if the number of members of the school council were increased to 37? Explain your reasoning.

D. Ms. Spencer needs 150 graphing calculators for her math students. Her budget allows $5,000 for calculators. She needs to know if she can buy what she needs at the discount store where calculators are on sale at 8 for $284.

   She writes the following statement:

   $$\frac{8}{284} = \frac{150}{x} \quad \text{or} \quad \frac{8}{284} = 150 \div x$$

   1. Use fact-family relationships to rewrite the proportion so that it is easier to find x.

   2. Solve the proportion, recording and explaining each of your steps.

   3. Is your method a general method that can be used to solve any proportion? Explain.

ACE Homework starts on page 55.

Applications

1. Jared and Pedro walk 1 mile in about 15 minutes. They can keep up this pace for several hours.

   a. About how far do they walk in 90 minutes?

   b. About how far do they walk in 65 minutes?

2. Swimming $\frac{1}{4}$ of a mile uses about the same number of Calories as running 1 mile.

   a. Gilda ran a 26-mile marathon. About how far would her sister have to swim to use the same number of Calories Gilda used during the marathon?

   b. Juan swims 5 miles a day. About how many miles would he have to run to use the same number of Calories used during his swim?

3. After testing many samples, an electric company determined that approximately 2 of every 1,000 light bulbs on the market are defective. Americans buy more than 1 billion light bulbs every year. Estimate how many of these bulbs are defective.

4. The organizers of an environmental conference order buttons for the participants. They pay $18 for 12 dozen buttons. Write and solve proportions to answer each question. Assume that price is proportional to the size of the order.

   a. How much do 4 dozen buttons cost?

   b. How much do 50 dozen buttons cost?

   c. How many dozens can the organizers buy for $27?

   d. How many dozens can the organizers buy for $63?

Homework Help Online
PHSchool.com
For: Help with Exercise 4
Web Code: ane-3404

5. Denzel makes 10 of his first 15 shots in a basketball free-throw contest. His success rate stays about the same for his next 100 free throws. Write and solve a proportion to answer each part. Round to the nearest whole number. Start each part with the original 10 of 15 free throws.

   a. About how many free throws does Denzel make in his next 60 attempts?

   b. About how many free throws does he make in his next 80 attempts?

   c. About how many attempts does Denzel take to make 30 free throws?

   d. About how many attempts does he take to make 45 free throws?

For Exercises 6–13, solve each equation.

6. $12.5 = 0.8x$ **7.** $\frac{x}{15} = \frac{20}{50}$ **8.** $\frac{x}{18} = 4.5$ **9.** $\frac{15.8}{x} = 0.7$

10. $\frac{5}{9} = \frac{12}{x}$ **11.** $245 = 0.25x$ **12.** $\frac{18}{x} = \frac{4.5}{1}$ **13.** $\frac{0.1}{48} = \frac{x}{960}$

14. Multiple Choice Middletown sponsors a two-day conference for selected middle-school students to study government. There are three middle schools in Middletown.

Suppose 20 student delegates will attend the conference. Each school should be represented fairly in relation to its population. How many should be selected from each school?

North Middle School
618 students

Central Middle School
378 students

South Middle School
204 students

   A. North: 10 delegates, Central: 8 delegates, South: 2 delegates

   B. North: 11 delegates, Central: 7 delegates, South: 2 delegates

   C. North: 6 delegates, Central: 3 delegates, South: 2 delegates

   D. North: 10 delegates, Central: 6 delegates, South: 4 delegates

Connections

For Exercises 15–17, use ratios, percents, fractions, or rates.

15. **Multiple Choice** Which cereal is the best buy?

   F. a 14-ounce box for $1.98   **G.** a 36-ounce box for $2.59

   H. a 1-ounce box for $0.15   **J.** a 72-ounce box for $5.25

16. Which is the better average: 10 of 15 free throws, or 8 of 10 free throws?

17. Which is the better home-run rate: two home runs per 60 times at bat, or five home runs per 120 times at bat?

18. A jar contains 150 marked beans. Scott takes several samples from the jar and gets the results shown.

Bean Samples

Number of Beans	25	50	75	100	150	200	250
Number of Marked Beans	3	12	13	17	27	38	52
Percent of Marked Beans	12%	▨	▨	▨	▨	▨	▨

   a. Copy and complete the table.

   b. Graph the data using (*number of beans, marked beans*) as data points. Describe the pattern of data points in your graph. What does the pattern tell you about the relationship between the number of beans in a sample and the number of marked beans you can expect to find?

19. **Multiple Choice** Ayanna is making a circular spinner to be used at the school carnival. She wants the spinner to be divided so that 30% of the area is blue, 20% is red, 15% is green, and 35% is yellow. Choose the spinner that fits the description.
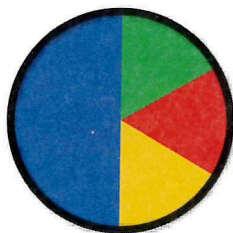
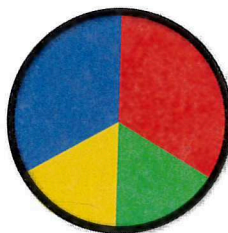**A.**
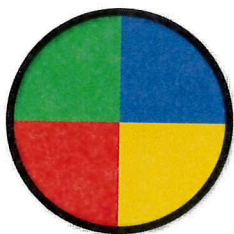
B.
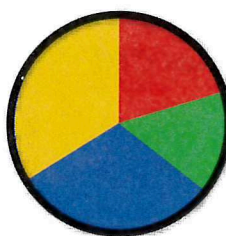
C.

D.

20. Hannah is making her own circular spinner. She makes the ratio of green to yellow 2 : 1, the ratio of red to yellow 3 : 1, and the ratio of blue to green 2 : 1. Make a sketch of her spinner.

21. a. Plot the points $(8, 6)$, $(8, 22)$, and $(24, 14)$ on grid paper. Connect them to form a triangle.

  b. Draw the triangle you get when you apply the rule $(0.5x, 0.5y)$ to the three points from part (a).

  c. How are lengths of corresponding sides in the triangles from parts (a) and (b) related?

  d. The area of the smaller triangle is what percent of the area of the larger triangle?

  e. The area of the larger triangle is what percent of the area of the smaller triangle?

22. The sketch shows two similar polygons.
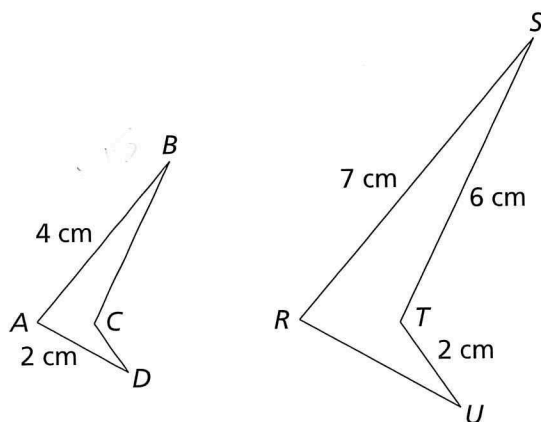
  a. What is the length of side BC?

  b. What is the length of side RU?

  c. What is the length of side CD?

23. To earn an Explorer Scout merit badge, Yoshi and Kai have the task of measuring the width of a river. Their report includes a diagram that shows their work.
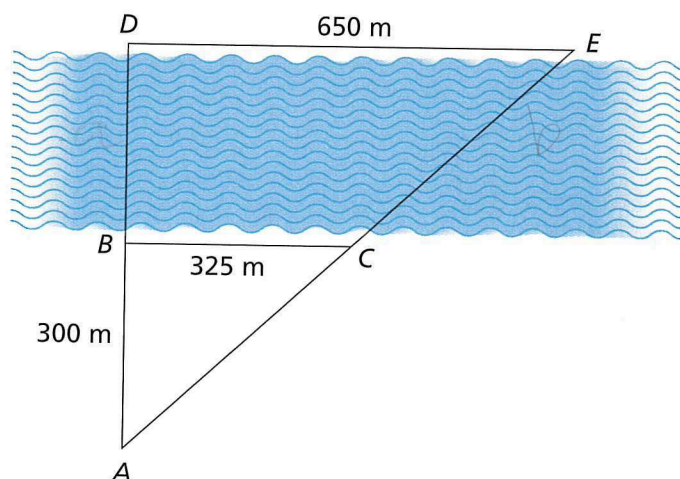
a. How do you think they came up with the lengths of the segments *AB*, *BC*, and *DE*?

b. How can they find the width of the river from segments *AB*, *BC*, and *DE*?

Extensions

24. Angela, a biologist, spends summers on an island in Alaska. For several summers she studied puffins. Two summers ago, Angela captured, tagged, and released 20 puffins. This past summer, she captured 50 puffins and found that 2 of them were tagged. Using Angela's findings, estimate the number of puffins on the island. Explain.

25. Rita wants to estimate the number of beans in a large jar. She takes out 100 beans and marks them. Then she returns them to the jar and mixes them with the unmarked beans. She then gathers some data by taking a sample of beans from the jar. Use her data to predict the number of beans in the jar.
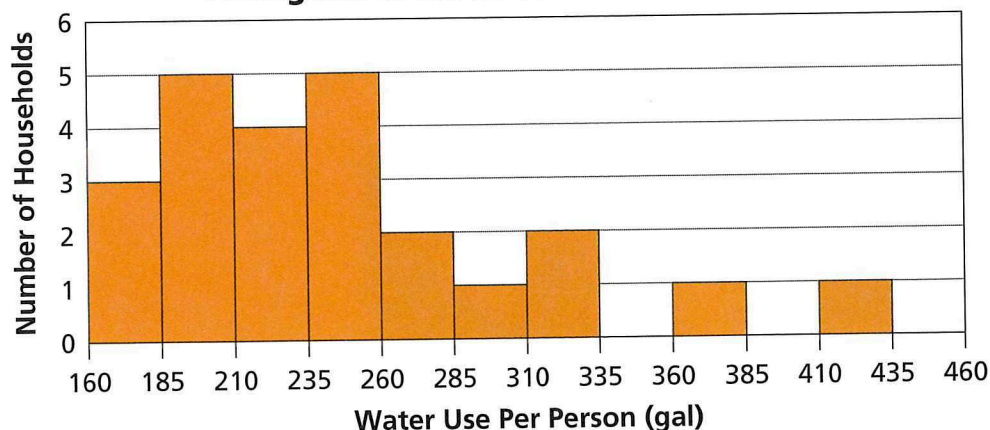
> **Sample**
>
> Number of marked beans: 2
> Beans in sample: 30

26. The two histograms below display information about gallons of water used per person in 24 households in a week.

**Histogram A: Water Use in 24 Households**
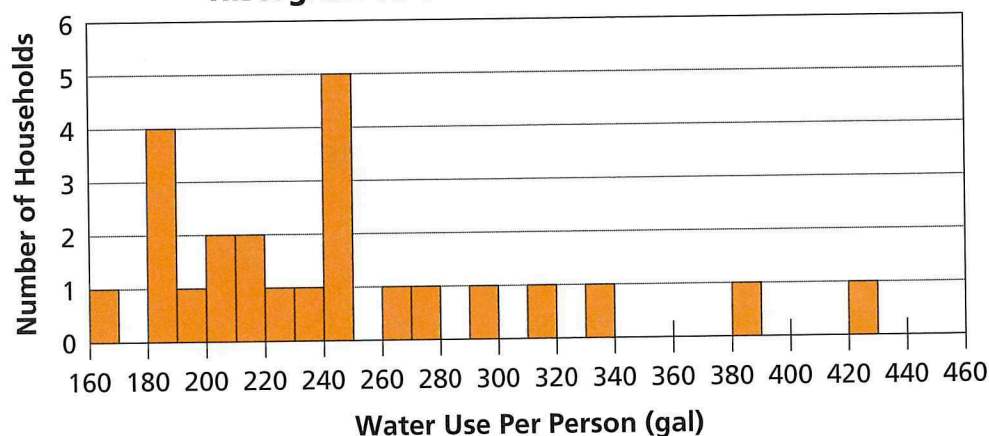

Histogram B: Water Use in 24 Households

a. Compare the two histograms and explain how they differ.

b. Where do the data seem to clump in Histograms A and B?

27. The picture at the right is drawn on a centimeter grid.

  a. On a grid made of larger squares than those shown here, draw a figure similar to this figure. What is the scale factor between the original figure and your drawing?

  b. Draw another figure similar to this one, but use a grid made of smaller squares than those shown here. What is the scale factor between the original and your drawing?

  c. Compare the perimeters and areas of the original figure and its copies in each case (enlargement and reduction of the figure). Explain how these values relate to the scale factor in each case.
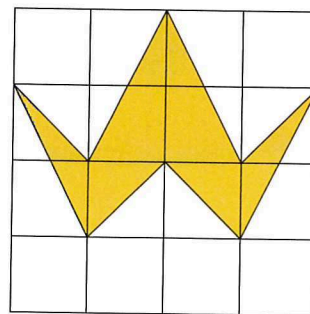
28. The people of the United States are represented in Congress, which is made up of the House of Representatives and the Senate.

  a. In the House of Representatives, the number of representatives from each state varies. From what you know about Congress, how is the number of representatives from each state determined?

  b. How is the number of senators from each state determined?

  c. Compare the two methods of determining representation in Congress. What are the advantages and disadvantages of these two forms of representation for states with large populations? How about for states with small populations?

Extensions
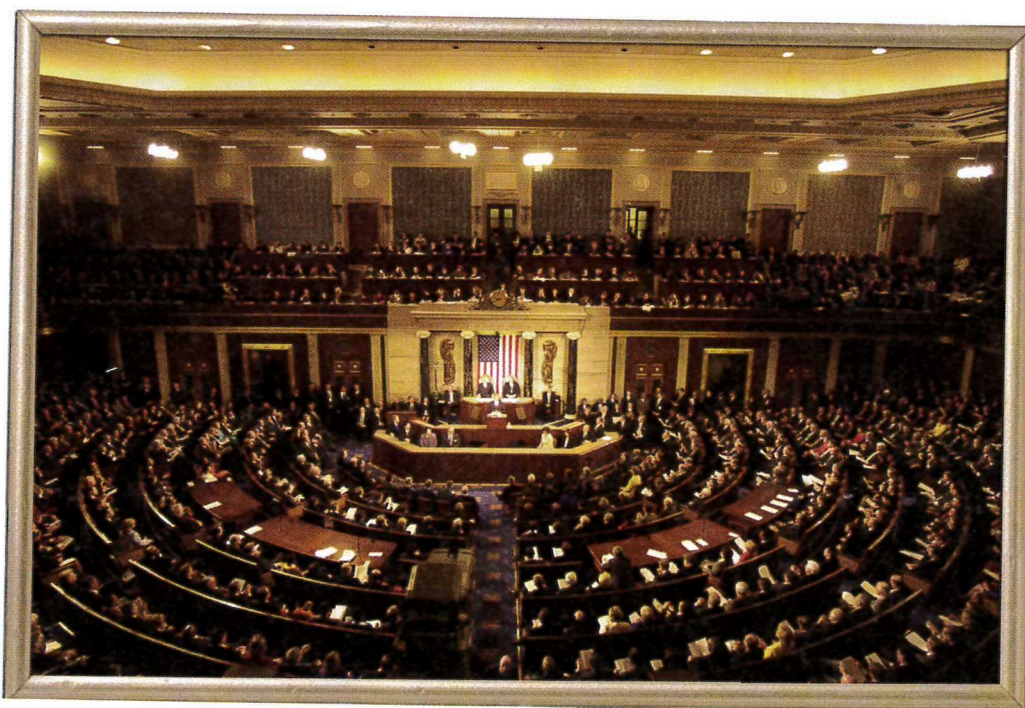
In this investigation, you used ratios and proportions to solve a variety of problems. You found that most of those problems can be expressed in proportions such as $\frac{a}{b} = \frac{c}{x}$ or $\frac{a}{b} = \frac{x}{c}$. The next questions will help you summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. For each situation, write a problem that can be solved using a proportion. Then solve your problem.

   a. The fraction of girls in grade seven is $\frac{3}{5}$.

   b. Bolda Cola sells at 5 for $3.

   c. Sora rides her bike at a speed of 12 miles per hour.

   d. A triangle is similar to another one with a scale factor of 1.5.

2. Write four different proportions for the problem you created in part (c). Show that the answer to the problem is the same no matter which proportion you use.

3. What procedures do you use to solve proportions such as those you wrote in Question 2?

Unit Project

Paper Pool

The unit project is a mathematical investigation of a game called Paper Pool. For a pool table, use grid paper rectangles like the one shown at the right. Each outside corner is a pocket where a "ball" could "fall."

Pocket D   Pocket C
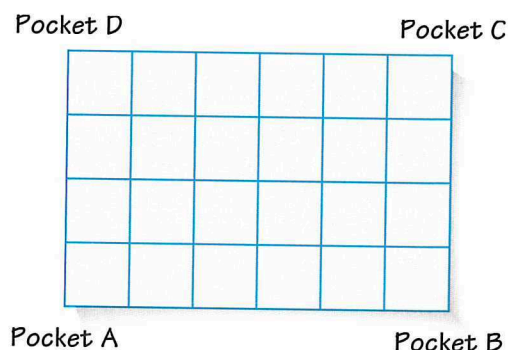
Pocket A   Pocket B

How to Play Paper Pool

- The ball always starts at Pocket A.
- To move the ball, "hit" it as if you were playing pool.
- The ball always moves on a 45° diagonal across the grid.
- When the ball hits a side of the table, it bounces off at a 45° angle and continues to move.
- If the ball moves to a corner, it falls into the pocket at that corner.

The dotted lines on the table at the right show the ball's path.

- The ball falls in Pocket D.
- There are five "hits," including the starting hit and the final hit.
- The dimensions of the table are 6 by 4 (always mention the horizontal length first).

Pocket D   "hit"   Pocket C
"hit"

"hit"

"hit"

Pocket A   "hit"   Pocket B
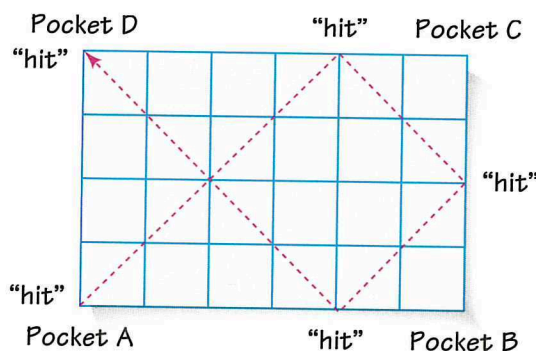
Part 1: Investigate Two Questions

Use the three Paper Pool labsheets to play the game. Try to find rules that tell you (1) the pocket where the ball will fall and (2) the number of hits along the way. Keep track of the dimensions because they may give you clues to a pattern.

Part 2: Write a Report

When you find some patterns and reach some conclusions, write a report that includes

- A list of the rules you found and an explanation of why you think they are correct
- Drawings of other grid paper tables that follow your rule
- Any tables, charts, or other tools that helped you find patterns
- Other patterns or ideas about Paper Pool

Extension Question

Can you predict the length of the ball's path on any size Paper Pool table? Each time the ball crosses a square, the length is 1 diagonal unit. Find the length of the ball's path in diagonal units for any set of dimensions.

active math online

For: Paper Pool Activity
Visit: PHSchool.com
Web Code: and-3000

Looking Back and Looking Ahead

The problems in this unit required you to compare measured quantities. You learned when it seems best to use subtraction, division, percents, rates, ratios, and proportions to make those comparisons. You developed a variety of strategies for writing and solving proportions. These strategies include writing equivalent ratios to scale a ratio up or down. You also learned to compute and reason with unit rates.

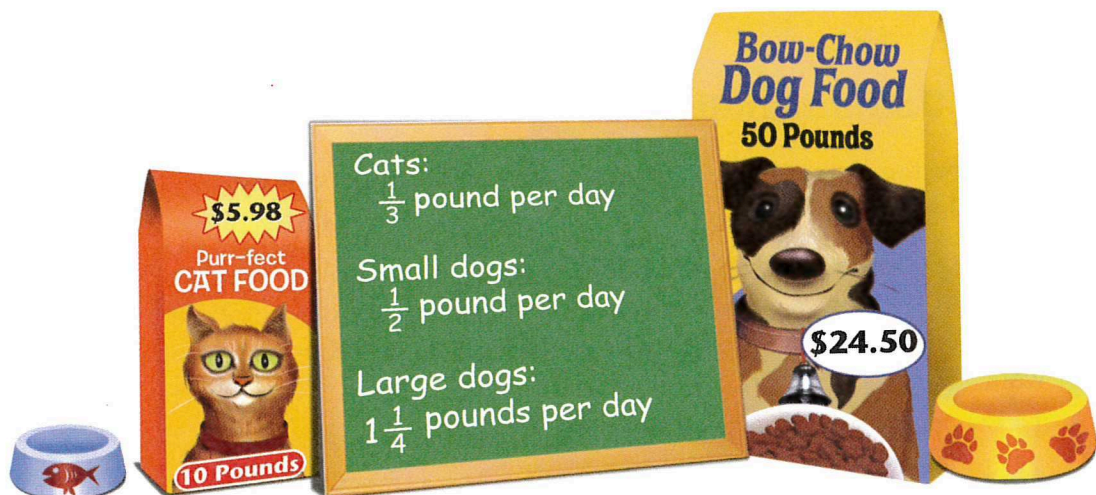## Use Your Understanding: Proportional Reasoning

Test your understanding of percents, rates, ratios, and proportions by solving the following problems.

1. There are 300 students in East Middle School. To plan transportation services for the new West Middle School, the school system surveyed East students. The survey asked whether students ride a bus to school or walk.

   - In Mr. Archer's homeroom, 20 students ride the bus and 15 students walk.

   - In Ms. Brown's homeroom, 14 students ride the bus and 9 students walk.

   - In Mr. Chavez's homeroom, 20 students ride the bus and the ratio of bus riders to walkers is 5 to 3.

   a. In what ways can you compare the number of students in Mr. Archer's homeroom who are bus riders to the number who are walkers? Which seems to be the best comparison statement?

   b. In what ways can you compare the numbers of bus riders and walkers in Ms. Brown's homeroom to those in Mr. Archer's homeroom? Again, which seems the best way to make the comparison?

   c. How many students in Mr. Chavez's homeroom walk to school?

d. Use the information from these three homerooms. About how many East Middle School students would you expect to walk to school? How many would you expect to ride a bus?

e. Suppose the new West Middle School will have 450 students and a ratio of bus riders to walkers that is about the same as that in East Middle School. About how many West students can be expected in each category?

2. The Purr & Woof Kennel buys food for animals that are boarded. The amounts of food eaten and the cost for food are shown below.

Cats: $\frac{1}{3}$ pound per day

Small dogs: $\frac{1}{2}$ pound per day

Large dogs: $1\frac{1}{4}$ pounds per day

a. Is cat food or dog food cheaper per pound?

b. Is it cheapest to feed a cat, a small dog, or a large dog?

c. On an average day, the kennel has 20 cats, 30 small dogs, and 20 large dogs. Which will last longer: a bag of cat food or a bag of dog food?

d. How many bags of dog food will be used in the month of January? How many bags of cat food will be used?

e. The owner finds a new store that sells Bow-Chow in 15 pound bags for $6.75 per bag. How much does that store charge for 50 pounds of Bow-Chow?

f. Which is a better buy on Bow-Chow: the original source or the new store?