Investigation 1

A First Look at Chance

Decisions, decisions, decisions! You make decisions every day. You choose what to wear, with whom to have lunch, what to do after school, and maybe what time to go to bed.

You make some decisions without even thinking. For example, you may automatically eat the same breakfast cereal each morning. You base other decisions on how you feel at a given time. If you are in the mood to laugh, you might decide to meet a friend with a good sense of humor.

To make some decisions, you consider the chance, or likelihood, that something will happen. You may listen to the weather forecast to decide whether you will wear a raincoat to school. In some cases, you may even let chance make a decision for you, such as when you roll a number cube to see who goes first in a game.

A number cube shows the numbers 1, 2, 3, 4, 5, and 6 on its faces.

1.1 Choosing Cereal

Getting Ready for Problem 1.1

- What are the chances of getting a 2 when you roll a number cube? Are you more likely to roll a 2 or a 6? How can you decide?
- The weather forecaster says the chance of rain tomorrow is 40%. What does this mean? Should you wear a raincoat?
- When you toss a coin, what are the chances of getting tails? If you toss seven tails in a row, are you more likely to get heads or tails on the next toss?

Kalvin always has cereal for breakfast. He likes Cocoa Blast cereal so much that he wants it every morning. Kalvin's mother wants him to eat Health Nut Flakes at least some mornings because it is more nutritious than Cocoa Blast.

Kalvin and his mother have found a fun way to choose which cereal he will have for breakfast. Each morning in June, Kalvin tosses a coin. If the coin lands on heads, he will have Cocoa Blast. If the coin lands on tails, he will have Health Nut Flakes.

Predict how many days in June Kalvin will eat Cocoa Blast.

Problem 1.1 Finding Probabilities With a Coin

A. 1. Conduct an experiment to test your prediction. Toss a coin 30 times (one for each day in June). Record your results in a table such as the one shown with 30 rows:

Coin Toss Results

Day	Result of Toss (H or T)	Number of Heads So Far	Fraction of Heads So Far	Percent of Heads So Far
1				
2				

2. As you add more data, what happens to the percent of tosses that are heads?

B. Work with your teacher and your classmates to combine the results from all the groups.

1. What percent of the total number of tosses for your class is heads?
2. As your class adds more data, what happens to the percent of tosses that are heads?
3. Based on what you found for June, how many times do you expect Kalvin to eat Cocoa Blast in July? Explain your reasoning.

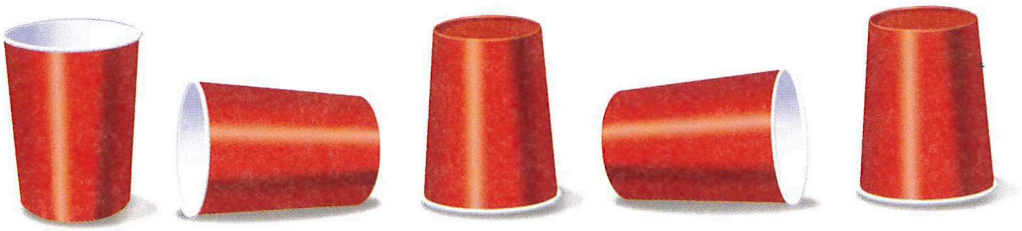
C. Kalvin's mother tells him that the chance of a coin showing heads when he tosses it is $\frac{1}{2}$. Does this mean that every time he tosses a coin twice he will get one head and one tail? Explain.

ACE **Homework starts on page 13.**

1.2 Tossing Paper Cups

Kalvin really loves Cocoa Blast. He wants to find something else to toss that will give him a better chance of eating the cereal each morning. He looks through a cupboard and finds a package of paper cups. He wonders if a paper cup is a good thing to toss.

Because Kalvin wants to eat Cocoa Blast cereal more of the time, he needs to determine if the cup lands in one position more often than another. If so, he will ask to toss a paper cup instead of a coin.



End **Side** **End** **Side** **End**

Which of the landing positions, end or side, should Kalvin use to represent Cocoa Blast? (Remember, he wants to eat Cocoa Blast as often as possible.)

Problem 1.2 Finding More Probabilities

A. Conduct an experiment to test your prediction about how a paper cup lands. Toss a paper cup 50 times. Make a table to record your data.

B. Use your results to answer the following questions:

1. For what fraction of your 50 tosses did the cup land on one of its ends? What percent is this?
2. For what fraction of your 50 tosses did the cup land on its side? What percent is this?
3. Do the landing positions *end* and *side* have the same chance of occurring? If not, which is more likely? Explain.
4. Which of the cup's landing positions should Kalvin use to represent Cocoa Blast? Explain your reasoning.

C. Combine the data from all the groups in your class. Based on these data, would you change your answers to Question B, parts (3) and (4), above? Explain.

D. Kalvin's mom agrees to let him use a cup to decide his cereal each morning. On the first morning, the cup lands on its end. On the second morning, it lands on its side. Kalvin says, "This cup isn't any better than the coin. It lands on an end 50% of the time!" Do you agree or disagree with Kalvin? Explain.

ACE Homework starts on page 13.

1.3 One More Try

In the last two problems, you conducted experiments and found the chances of particular results. You represented these chances as fractions or percents. The mathematical word for chance is **probability**. A probability that you find by conducting an experiment and collecting data is called an **experimental probability**.

Suppose you toss a paper cup 50 times, and it lands on its side 31 times. Based on these data, the experimental probability that the cup will land on its side is $\frac{31}{50}$. Each toss of the cup is called a *trial*.

Use the ratio below to find experimental probability.

$$\frac{\text{number of favorable trials}}{\text{total number of trials}}$$

Favorable trials are the trials in which the desired result occurs. To find the probability of a cup landing on its side, count each time the cup lands on its side as a favorable trial.

You can write "the probability of the cup landing on its side" as $P(\text{side})$. In the experiment just described,

$$P(\text{side}) = \frac{\text{number of times cup landed on its side}}{\text{number of times cup was tossed}} = \frac{31}{50}.$$

Kalvin has come up with one more way to use probability to decide his breakfast cereal. This time, he tosses two coins.

- If the coins match, he gets to eat Cocoa Blast.

Match

Match

- If the coins do not match, he eats Health Nut Flakes.

No Match

Suppose his mother agrees to let him use this method. How many days in June do you think Kalvin will eat Cocoa Blast?

Problem 1.3 Finding Experimental Probabilities

A. **1.** Conduct an experiment by tossing a pair of coins 30 times. Keep track of the number of times the coins *match* and the number of times a *no-match* occurs.

2. Based on your data, what is the experimental probability of getting a match? Of getting a no-match?

B. Combine your data with your classmates' data.

1. Find the experimental probabilities for the combined data. Compare these probabilities with the probabilities in Question A.

2. Based on the class data, do you think a match and a no-match have the same chance of occurring? Explain.

C. Think about the possible results when you toss two coins.

1. In how many ways can a match occur?

2. In how many ways can a no-match occur?

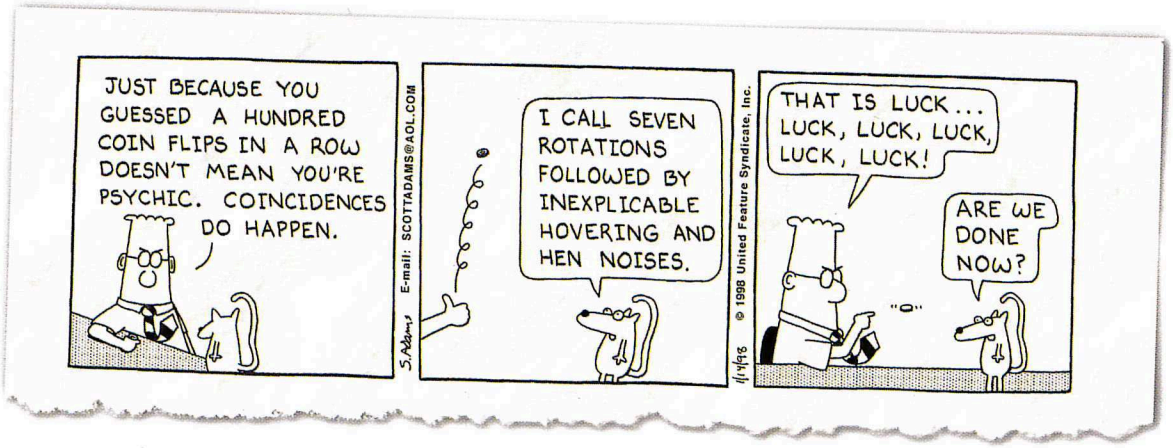
3. Based on the number of ways each result can occur, do a match and a no-match have the same chance of occurring? Explain.

D. Kalvin's friend Asta suggests that he toss a thumbtack. If it lands on its side, he eats Cocoa Blast. If it lands on its head, he eats Health Nut Flakes. She says they must first experiment to find the probabilities involved. Asta does 11 tosses. Kalvin does 50 tosses. Here are the probabilities they find based on their experiments:

Asta: $P(\text{heads}) = \frac{6}{11}$ Kalvin: $P(\text{heads}) = \frac{13}{50}$

Which result do you think better predicts the thumbtack landing on its head when tossed? Explain.

ACE Homework starts on page 13.



1.4 Analyzing Events

Kalvin finds a coin near a railroad track. It looks flat and a little bent, so he guesses it has been run over by a train. He decides to use this unusual coin to choose his breakfast cereal during November. By the end of the month, he has had Health Nut Flakes only seven times. His mother is suspicious of the coin. She wonders if the coin is fair.

- Why do you think Kalvin's mother is suspicious of the coin?
- What do you think it means for a coin to be "fair"?

Kalvin's mother explains why she is suspicious. "With a fair coin, heads and tails are **equally likely.** This means that you have the same chance of getting heads as tails." Kalvin is not sure what his mother means by "equally likely," so she uses an example to help explain.

"Suppose each person in our family writes his or her name on a card and puts the card in a hat. If you mix up the cards and pull one out, each name is equally likely to be picked. But suppose I put my name in the hat ten times. Then, the names are not equally likely to be picked. My name has a greater chance of being chosen."

Problem 1.4 Understanding Equally Likely Events

A. The table below lists several actions and possible results. In each case, decide whether the possible results are equally likely and explain. For actions 5–7, start by listing all the possible results.

Action	Possible Results
1. You toss an empty juice can.	The can lands on its side, the can lands upside down, or the can lands right side up.
2. A baby is born.	The baby is a boy or the baby is a girl.

3. A baby is born.	The baby is right-handed or the baby is left-handed.
4. The Pittsburgh Steelers play a football game.	The Steelers win, the Steelers lose, or the Steelers tie.
5. You roll a six-sided number cube.	_____
6. You guess an answer on a true/false test.	_____
7. In basketball, you attempt a free throw.	_____

B. For which of the actions in Question A did you find the results to be equally likely? Does this mean that the probability of each result is $\frac{1}{2}$ (or 50%)? Explain.

C. Describe an action for which the results are equally likely. Then, describe an action for which the results are *not* equally likely.

ACE **Homework starts on page 13.**

Applications

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1. a. Miki tosses a coin 50 times and the coin shows heads 28 times. What fraction of the 50 tosses is heads? What percent is this?

b. Suppose the coin is fair, and Miki tosses it 500 times. About how many times can she expect it to show heads? Explain your reasoning.

2. Suppose Kalvin tosses a coin to determine his breakfast cereal every day. He starts on his twelfth birthday and continues until his eighteenth birthday. About how many times would you expect him to eat Cocoa Blast cereal?

3. Kalvin tosses a coin five days in a row and gets tails every time. Do you think there is something wrong with the coin? How can you find out?

4. Len tosses a coin three times. The coin shows heads every time. What are the chances the coin shows tails on the next toss? Explain.

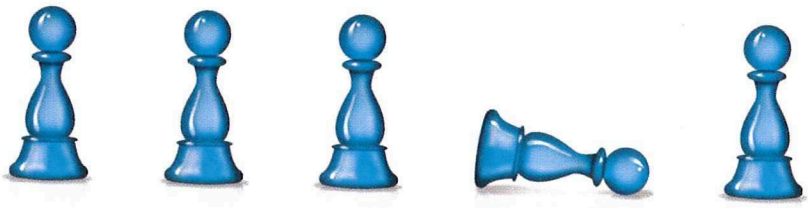
5. Is it possible to toss a coin 20 times and have it land heads up 20 times? Is this likely to happen? Explain.

6. Kalvin tosses a paper cup once each day for a year to determine his breakfast cereal. Use your results from Problem 1.2 to answer the following.

a. How many times do you expect the cup to land on its side? On one of its ends?

b. How many times a month do you expect Kalvin to eat Cocoa Blast? How many times a year? Explain.

7. Dawn tosses a pawn from her chess set five times. It lands on its base four times and on its side only once.



Andre tosses the same pawn 100 times. It lands on its base 28 times and on its side 72 times. Based on their data, if you toss the pawn one more time, is it more likely to land on its base or its side? Why?

8. Kalvin flips a small paper cup 50 times and a large paper cup 30 times. The table below displays the results of his experiments. Based on this data, should he use the small cup or the large cup to determine his breakfast each morning? Explain.

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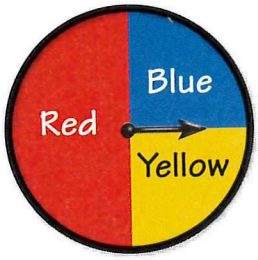
Paper Cup Toss Results

Where Cup Lands	Small Paper Cup	Large Paper Cup
Side	39 times	22 times
One of Its Ends	11 times	8 times

9. Kalvin's sister Kyla finds yet another way for him to pick his breakfast. She places one blue marble and one red marble in each of two bags. She says that each morning he can choose one marble from each bag. If the marbles are the same color, he eats Cocoa Blast. If not, he eats Health Nut Flakes. Explain how selecting one marble from each of the two bags and tossing two coins are similar.

10. Brooke and Jake have to decide who will take out the garbage. Jake suggests they toss two coins. If at least one head comes up, Brooke takes out the garbage. If no heads come up, Jake takes out the garbage. Should Brooke agree to Jake's proposal? Why or why not?

For Exercises 11–15, decide whether the possible results are equally likely. Explain.

Action	Possible Results
11. Your phone rings at 9:00 P.M.	The caller is your best friend, the caller is a relative, or the caller is someone else.
12. You check the temperature in your area tomorrow morning.	The temperature is 30°F or higher, or the temperature is below 30°F.
13. You spin the pointer once. 	The pointer lands on yellow, the pointer lands on red, or the pointer lands on blue.
14. You find out how many car accidents occurred in your city or town yesterday.	There were fewer than five accidents, there were exactly five accidents, or there were more than five accidents.
15. You choose a card from a standard deck of playing cards (with no jokers).	The card is a spade, the card is a heart, the card is a diamond, or the card is a club.

For Exercises 16 and 17, first list all the possible results for each action. Then, decide whether the results are equally likely.

16. You choose a block from a bag containing one red block, three blue blocks, and one green block.

17. You try to steal second base during a baseball game.

18. For parts (a)–(f), give an example of a result that would have a probability near the percent given.

a. 0% **b.** 25% **c.** 50%

d. 75% **e.** 80% **f.** 100%

Connections

19. Colby rolls a number cube several times. She records the result of each roll and organizes her data in the table below.

Number Cube Results

Number	Times the Number is Rolled
1	𝍸 |
2	𝍸 ||||
3	𝍸 |
4	𝍸 |||
5	𝍸 𝍸 |
6	𝍸 𝍸

a. What fraction of the rolls are 2's? What percent is this?

b. What fraction of the rolls are odd numbers? What percent is this?

c. What percent of the rolls is greater than 3?

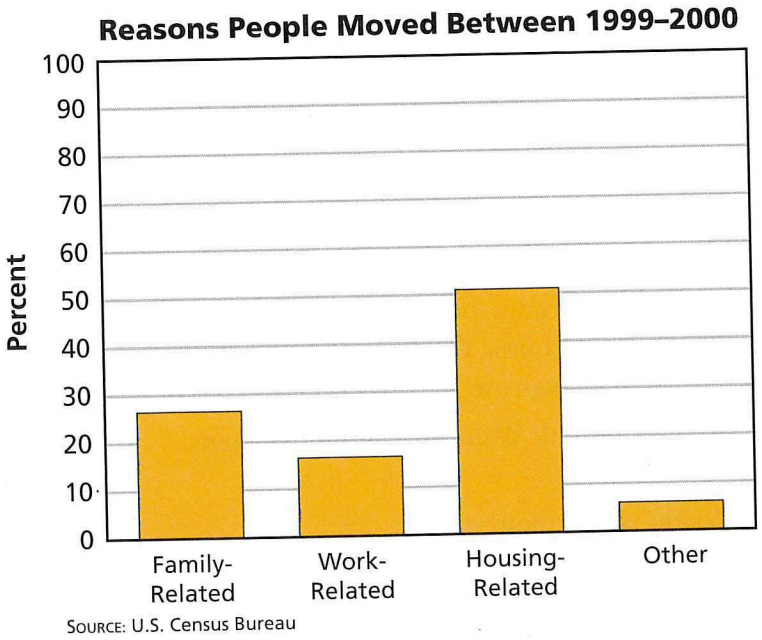
d. Suppose Colby rolls the number cube 100 times. About how many times can she expect to roll a 2? Explain.

e. If Colby rolls the number cube 1,000 times, about how many times can she expect to roll an odd number? Explain.

20. For each pair of fractions, find a fraction between the two fractions.

a. $\frac{1}{10}$ and $\frac{8}{25}$ **b.** $\frac{3}{8}$ and $\frac{11}{40}$

For Exercises 21–23, use the bar graph below.



SOURCE: U.S. Census Bureau

21. Multiple Choice Suppose 41,642 people moved. About how many of these people moved for family-related reasons?

A. 28 **B.** 11,000 **C.** 21,000 **D.** 31,000

22. Multiple Choice About what fraction of the people represented in the chart moved for reasons other than work-related, housing-related, or family-related?

F. $\frac{6}{10}$ **G.** $\frac{6}{100}$ **H.** $\frac{52}{100}$ **J.** $\frac{94}{100}$

23. Multiple Choice Suppose 41,642 people moved. About how many of these people moved for housing-related reasons?

A. 52 **B.** 11,000 **C.** 21,000 **D.** 31,000

24. Suppose you write each factor of 42 on pieces of paper and put them in a bag. You shake the bag and then choose one piece of paper from the bag. Find the probability of choosing a factor that is

a. an even number.

b. a prime number.

25. Weather forecasters often use percents to give probabilities in their forecasts. For example, a forecaster might say that there is a 50% chance of rain tomorrow. For the forecasts below, change the fractional probabilities to percents.

a. The probability that it will rain tomorrow is $\frac{2}{5}$.

b. The probability that it will snow Monday is $\frac{3}{10}$.

c. The probability that it will be cloudy this weekend is $\frac{3}{5}$.

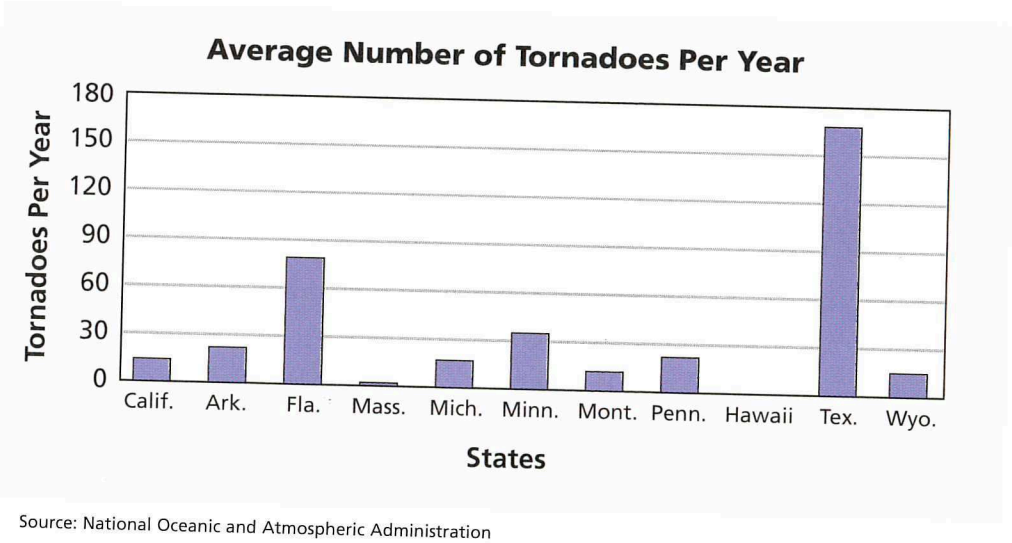
26. Waldo, the meteorologist from WARM radio, boasts that he is the best weather predictor in Sunspot, South Carolina. On Monday, Waldo says, "There is only a 10% chance of rain tomorrow!"

a. Ask at least two adults what they think Waldo's statement means. Write down their explanations.

b. Explain what you think Waldo's statement means.

c. If it rains on Tuesday, is Waldo wrong? Why or why not?

For Exercises 27–30, use this graph, which shows the average number of tornadoes per year in several states.



Source: National Oceanic and Atmospheric Administration

27. In an average year, is a tornado equally likely to occur in California as in Florida? Explain your reasoning.

28. In an average year, is a tornado equally likely to occur in Arkansas as in Pennsylvania?

29. In an average year, is a tornado equally likely to occur in Massachusetts as in Texas?

30. Based on these data, is a person living in Montana more likely to experience a tornado than a person living in Massachusetts? Explain.

Extensions

31. Monday is the first day Kalvin tosses a coin to determine his cereal. During the first five days, he has Cocoa Blast only twice. One possible pattern of Kalvin's coin tosses is shown.

Coin Toss Results

Monday	Tuesday	Wednesday	Thursday	Friday
H	H	T	T	T

Find every way Kalvin can toss the coin during the week and have Cocoa Blast cereal twice. Explain how you know that you found every possible way.

32. Yolanda watches a carnival game in which a paper cup is tossed. It costs \$1 to play the game. If the cup lands upright, the player wins \$5. The cup is tossed 50 times. It lands on its side 32 times, upside down 13 times, and upright 5 times.

a. If Yolanda plays the game ten times, about how many times can she expect to win? How many times can she expect to lose?

b. Do you expect her to have more or less money at the end of ten games? Why?

Mathematical Reflections 1

In this investigation, you conducted experiments with coins and paper cups. You used fractions and percents to express the chances, or probabilities, that certain results would occur. You also considered several actions and determined whether the possible results were equally likely. These questions will help you summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. How do you find the experimental probability that a particular result will occur? Why is it called the experimental probability?
2. In an experiment, are 30 trials as good as 500 trials to predict the chances of a result? Explain.
3. What does it mean for results to be equally likely?