Investigation 2

Experimental and Theoretical Probability

In the last investigation, you collected the results of many coin tosses. You found that the experimental probability of a coin landing on heads is $\frac{1}{2}$ (or very close to $\frac{1}{2}$).

The results of the coin-tossing experiment probably didn't surprise you. You already knew that the two possible results, heads and tails, are equally likely. In fact, you can find the probability of tossing heads by examining the possible results rather than by experimenting. There are two equally likely results. Because one of the results is heads, the probability of tossing heads is 1 of 2, or $\frac{1}{2}$.

The individual results of an action or event are called **outcomes.** The coin-tossing experiment had two outcomes, heads and tails. A probability calculated by examining outcomes, rather than by experimenting, is a **theoretical probability.**

When the outcomes of an action or event are equally likely, you can use the ratio below to find the theoretical probability.

$$\frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

Favorable outcomes are the outcomes in which you are interested.

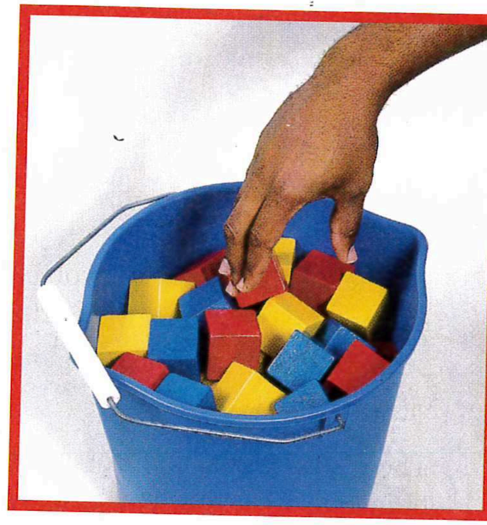
You can write the theoretical probability of tossing heads as $P(\text{heads})$. So,

$$P(\text{heads}) = \frac{\text{number of ways heads can occur}}{\text{number of outcomes}} = \frac{1}{2}.$$

In this investigation, you will explore some other situations in which probabilities are found both by experimenting and by analyzing the possible outcomes.

2.1 Predicting to Win

In the last 5 minutes of the *Gee Whiz Everyone Wins!* game show, all the members of the audience are called to the stage. They each choose a block at *random* from a bucket containing an unknown number of red, yellow, and blue blocks. Each block has the same size and shape. Before choosing, each contestant predicts the color of his or her block. If the prediction is correct, the contestant wins. After each selection, the block is put back into the bucket.



What do you think random *means? Suppose you are a member of the audience. Would you rather be called to the stage first or last? Why?*

Problem 2.1 Finding Theoretical Probabilities

A. 1. Play the block-guessing game with your class. Keep a record of the number of times a color is chosen. Play the game until you think you can predict the chances of each color being chosen.

2. Based on the data you collect during the game, find the experimental probabilities of choosing red, choosing yellow, and choosing blue.

B. 1. After you look in the bucket, find the fraction of the blocks that are red, the fraction that are yellow, and the fraction that are blue. These are the theoretical probabilities.

2. How do the theoretical probabilities compare to the experimental probabilities in Question A?

3. What is the sum of the theoretical probabilities in Question B, part (1)?

C. 1. Does each block have an equally likely chance of being chosen? Explain.

2. Does each color have an equally likely chance of being chosen? Explain.

D. Which person has the advantage—the first person to choose from the bucket or the last person? Explain.

ACE **Homework starts on page 28.**

2.2 Exploring Probabilities

In the next problem set, you will discover some interesting facts about probabilities.

Problem 2.2 Exploring Probabilities

A. A bag contains two yellow marbles, four blue marbles, and six red marbles. You choose a marble from the bag at random.

1. What is the probability the marble is yellow? The probability it is blue? The probability it is red?
2. What is the sum of the probabilities from part (1)?
3. What color is the marble most likely to be?
4. What is the probability the marble is *not* blue?
5. What is the probability the marble is either red or yellow?
6. What is the probability the marble is white?
7. Mary says the probability the marble is blue is $\frac{12}{4}$. Anne says $\frac{12}{4}$ is impossible. Who is correct? Explain your reasoning.

B. Suppose the bag in Question A has twice as many marbles of each color. Do the probabilities change? Explain.

C. How many blue marbles do you add to the bag in Question A to have the probability of choosing a blue marble equal to $\frac{1}{2}$?

D. A bag contains several marbles. Each marble is either red, white, or blue. The probability of choosing a red marble is $\frac{1}{3}$, and the probability of choosing a white marble is $\frac{1}{6}$.

1. What is the probability of choosing a blue marble? Explain.
2. What is the least number of marbles that can be in the bag? Explain. Suppose the bag contains the least number of marbles. How many of each color does the bag contain?
3. Can the bag contain 48 marbles? If so, how many of each color would it contain?
4. Suppose the bag contains 8 red marbles and 4 white marbles. How many blue marbles does it contain?

ACE Homework starts on page 28.

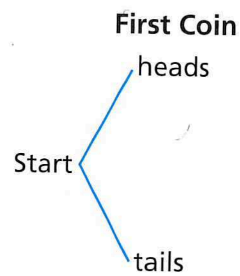
2.3 Winning the Bonus Prize

To find the theoretical probability of a result, you need to count all the possible outcomes. In some situations, such as when you toss a coin or roll a number cube, it is easy to count the outcomes. In other situations, it can be difficult. One way to find (or count) all the possible outcomes is to make an organized list. Here is an organized list of all the possible outcomes of tossing two coins.

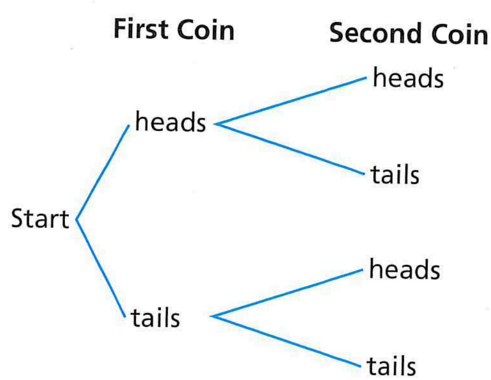
First Coin	Second Coin	Outcome
heads	heads	heads-heads
heads	tails	heads-tails
tails	heads	tails-heads
tails	tails	tails-tails

Another way to find all possible outcomes is to make a **tree diagram.** A tree diagram is a diagram that shows all the possible outcomes of an event. The steps for making a counting tree for tossing two coins are shown below.

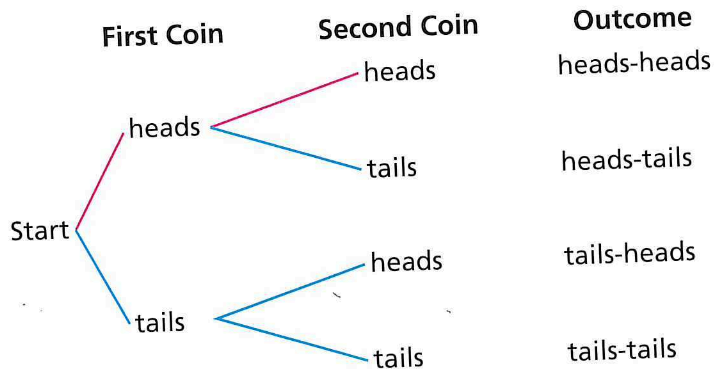
Step 1 Label a starting point. Make a branch from the starting point for each possible result for the first coin.



Step 2 Make a branch from each of the results for the first coin to show the possible results for the second coin.



Step 3 When you follow the paths from left to right, you can find all the possible outcomes of tossing two coins. For example, the path shown in red represents the outcome heads-heads.



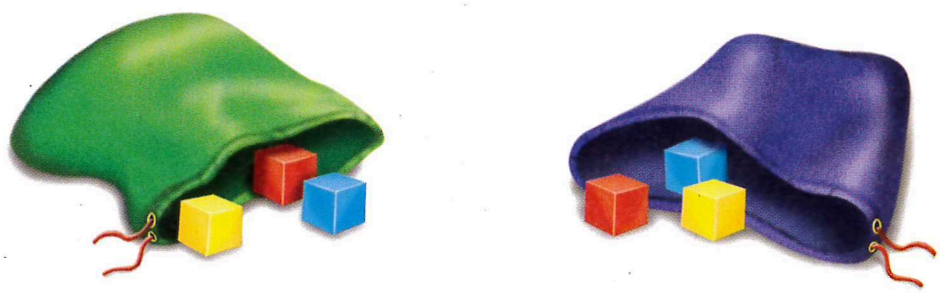
Both the organized list and the tree diagram show that there are four possible outcomes when you toss two coins. The outcomes are equally likely, so the probability of each outcome is $\frac{1}{4}$.

$$P(\text{heads, heads}) = \frac{1}{4}$$
$$P(\text{heads, tails}) = \frac{1}{4}$$
$$P(\text{tails, heads}) = \frac{1}{4}$$
$$P(\text{tails, tails}) = \frac{1}{4}$$

If you toss two coins, what is the probability that the coins will match?

What is the probability they won't match?

All the winners from the *Gee Whiz Everyone Wins!* game show have the opportunity to compete for a bonus prize. Each winner chooses one block from each of two bags. Both bags contain one red, one yellow, and one blue block. The contestant must predict which color she or he will choose from each of the two bags. If the prediction is correct, the contestant wins a $10,000 bonus prize!



What are the contestant's chances of winning this game?

Problem 2.3 Using Strategies to Find Theoretical Probabilities

A. **1.** Conduct an experiment with 36 trials for the situation above. Record the pairs of colors that you choose.

2. Find the experimental probability of choosing each possible pair of colors.

3. If you combined your data with the data collected by your classmates, would your answer to part (1) change? Explain.

B. **1.** List all the possible pairs that can be chosen. Are these outcomes equally likely? Explain your reasoning.

2. Find the theoretical probability of choosing each pair of blocks.

3. Does a contestant have a chance to win the bonus prize? Is it likely a contestant will win the bonus prize? Explain.

4. If you play this game 18 times, about how many times do you expect to win?

C. How do the theoretical probabilities compare with your experimental probabilities? Explain any differences.

ACE **Homework starts on page 28.**

2.4 Pondering Possible and Probable

Santo and Tevy are playing a coin-tossing game. To play the game, they take turns tossing three coins. If all three coins match, Santo wins. Otherwise, Tevy wins. Both players have won the game several times, but Tevy seems to be winning more often. Santo thinks the game is unfair.

Do you think this game is fair?

Problem 2.4 Pondering Possible and Probable

A. **1.** How many possible outcomes are there when you toss three coins? Show all your work. Are the outcomes equally likely?

2. What is the theoretical probability that the three coins will match?

3. What is the theoretical probability that exactly two coins will match?

4. Is this a fair game? Explain your reasoning.

B. If you tossed three coins 24 times, how many times would you expect two coins to match?

C. Santo said, "It is *possible* to toss three matching coins." Tevy replied, "Yes, but is it *probable*?" What do you think each boy meant?

ACE **Homework starts on page 28.**

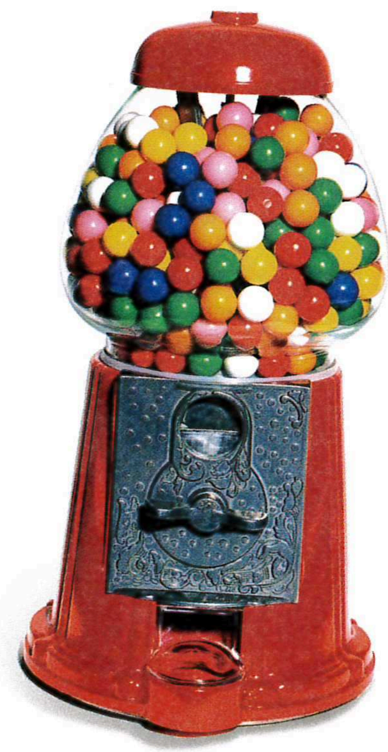
Applications Connections Extensions

ACE

Applications

1. A bucket contains one green block, one red block, and two yellow blocks. You choose one block from the bucket.
   a. Find the theoretical probability that you will choose each color.
      $P(\text{green}) = ■$ $P(\text{yellow}) = ■$ $P(\text{red}) = ■$
   b. Find the sum of the probabilities in part (a).
   c. What is the probability that you will *not* choose a red block? Explain how you found your answer.
   d. What is the sum of the probability of choosing a red block and the probability of *not* choosing a red block?

2. A bubble-gum machine contains 25 gumballs. There are 12 green, 6 purple, 2 orange, and 5 yellow gumballs.
   a. Find each theoretical probability.
      $P(\text{green}) = ■$ $P(\text{purple}) = ■$
      $P(\text{orange}) = ■$ $P(\text{yellow}) = ■$
   b. Find the sum.
      $P(\text{green}) + P(\text{purple}) + P(\text{orange}) + P(\text{yellow}) = ■$
   c. Write each of the probabilities in part (a) as a percent.
      $P(\text{green}) = ■$ $P(\text{purple}) = ■$
      $P(\text{orange}) = ■$ $P(\text{yellow}) = ■$
   d. What is the sum of all the probabilities as a percent?
   e. What do you think the sum of the probabilities for all the possible outcomes must be for any situation? Explain.



Applications

3. A bag contains two white blocks, one red block, and three purple blocks. You choose one block from the bag.

a. Find each probability.

$P(\text{white}) = \blacksquare$ $P(\text{red}) = \blacksquare$ $P(\text{purple}) = \blacksquare$

b. What is the probability of *not* choosing a white block? Explain how you found your answer.

c. Suppose the number of blocks of each color is doubled. What happens to the probability of choosing each color?

d. Suppose you add two more blocks of each color. What happens to the probability of choosing each color?

e. How many blocks of which colors should you add to the original bag to make the probability of choosing a red block equal to $\frac{1}{2}$?

4. A bag contains exactly three blue blocks. You choose a block at random. Find each probability.

a. $P(\text{blue})$ **b.** $P(\textit{not}\ \text{blue})$ **c.** $P(\text{yellow})$

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5. A bag contains several marbles. Some are red, some are white, and some are blue. You count the marbles and find the theoretical probability of choosing a red marble is $\frac{1}{5}$. You also find the theoretical probability of choosing a white marble is $\frac{3}{10}$.

a. What is the least number of marbles that can be in the bag?

b. Can the bag contain 60 marbles? If so, how many of each color does it contain?

c. If the bag contains 4 red marbles and 6 white marbles, how many blue marbles does it contain?

d. How can you find the probability of choosing a blue marble?

6. Decide whether each statement is true or false. Justify your answers.

a. The probability of an outcome can be 0.

b. The probability of an outcome can be 1.

c. The probability of an outcome can be greater than 1.

7. Melissa is designing a birthday card for her sister. She has a blue, a yellow, a pink, and a green sheet of paper. She also has a black, a red, and a purple marker. Suppose Melissa chooses one sheet of paper and one marker at random.

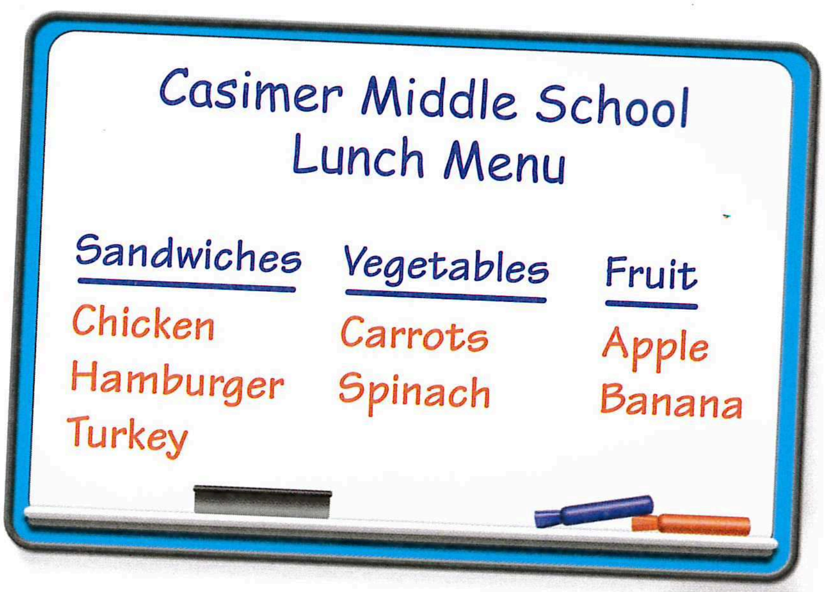
   a. Make a tree diagram to find all the possible color combinations.

   b. What is the probability that Melissa chooses pink paper and a red marker?

   c. What is the probability that Melissa chooses blue paper? What is the probability she does *not* choose blue paper?

   d. What is the probability that she chooses a purple marker?

8. Lunch at Casimer Middle School consists of a sandwich, a vegetable, and a fruit. Today there is an equal number of each type of sandwich, vegetable, and fruit. The students don't know what lunch they will get. Sol's favorite lunch is a chicken sandwich, carrots, and a banana.



   a. Make a tree diagram to determine how many different lunches are possible. List all the possible outcomes.

   b. What is the probability that Sol gets his favorite lunch? Explain your reasoning.

   c. What is the probability that Sol gets at least one of his favorite lunch items? Explain.

9. Suppose you spin the pointer of the spinner at the right once and roll the number cube. (The numbers on the cube are 1, 2, 3, 4, 5, and 6.)

a. Make a tree diagram of the possible outcomes of a spin of the pointer and a roll of the number cube.

b. What is the probability that you get a 2 on both the spinner and the number cube? Explain your reasoning.

c. What is the probability that you get a factor of 2 on both the spinner and the number cube?

d. What is the probability that you get a multiple of 2 on both the number cube and the spinner?

10. Patricia and Jean design a coin-tossing game. Patricia suggests tossing three coins. Jean says they can toss one coin three times. Are the outcomes different for the two situations? Explain.

11. Pietro and Eva are playing a game in which they toss a coin three times. Eva gets a point if *no* two consecutive toss results match (as in H-T-H). Pietro gets a point if exactly two consecutive toss results match (as in H-H-T). The first player to get 10 points wins. Is this a fair game? Explain. If it is not a fair game, change the rules to make it fair.

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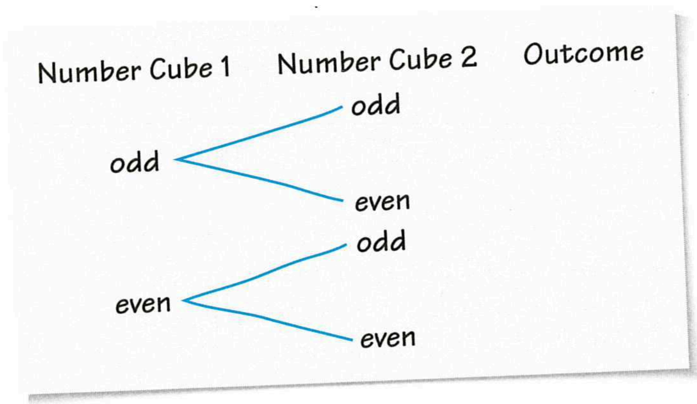
12. Silvia and Juanita are designing a game. In the game, you toss two number cubes and consider whether the sum of the two numbers is odd or even. They make a tree diagram of possible outcomes.



a. List all the outcomes.

b. Design rules for a two-player game that is fair.

c. Design rules for a two-player game that is not fair.

d. How is this situation similar to tossing two coins and seeing if the coins match or don't match?

Connections

13. Find numbers that make each sentence true.

a. $\frac{1}{8} = \frac{\blacksquare}{32} = \frac{5}{\blacksquare}$ **b.** $\frac{3}{7} = \frac{\blacksquare}{21} = \frac{6}{\blacksquare}$ **c.** $\frac{6}{20} = \frac{\blacksquare}{5} = \frac{12}{\blacksquare}$

14. Which of the following sums is equal to 1?

a. $\frac{1}{6} + \frac{3}{6} + \frac{2}{6}$ **b.** $\frac{4}{18} + \frac{1}{9} + \frac{2}{3}$ **c.** $\frac{1}{5} + \frac{1}{3} + \frac{1}{5}$

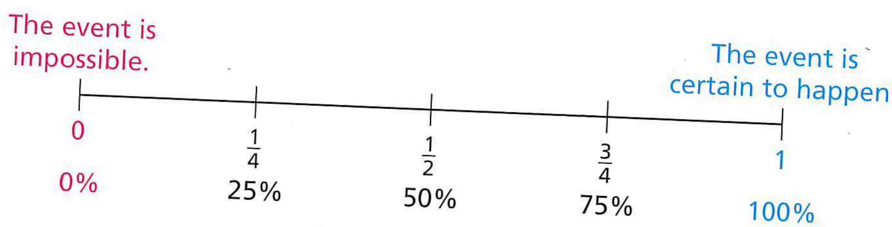
15. From Question 14, choose a sum equal to 1. Describe a situation whose events have a theoretical probability that can be represented by the sum.

16. Kara and Bly both perform the same experiment in math class. Kara gets a probability of $\frac{125}{300}$ and Bly gets a probability of $\frac{108}{320}$.

a. Whose experimental probability is closer to the theoretical probability of $\frac{1}{3}$? Explain your reasoning.

b. Give two possible experiments that Kara and Bly can do that have a theoretical probability of $\frac{1}{3}$.

For Exercises 17–24, estimate the probability that the given event occurs. Any probability must be between 0 and 1 (or 0% and 100%). If an event is impossible, the probability it will occur is 0, or 0%. If an event is certain to happen, the probability it will occur is 1, or 100%.



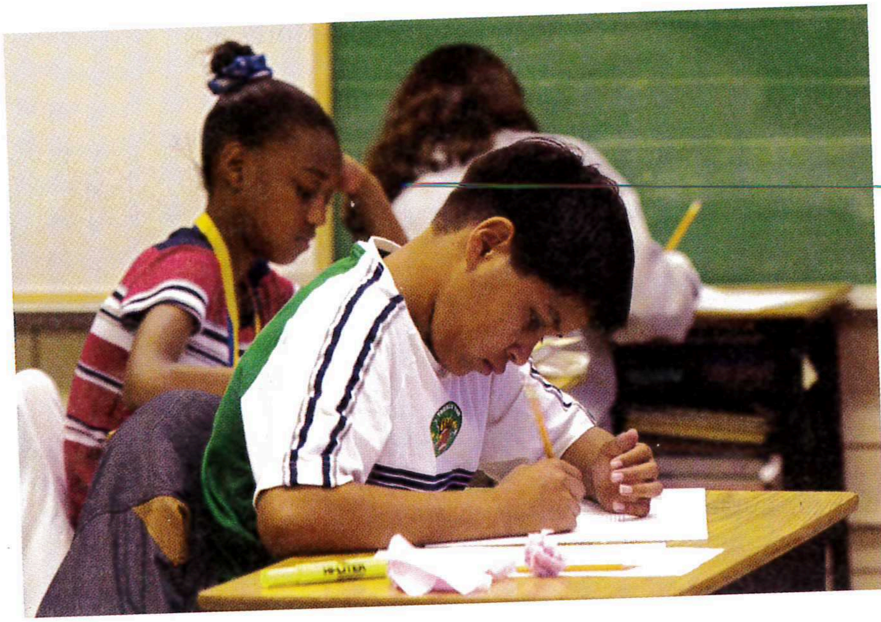
Sample You watch television tonight.

I watch some television every night, unless I have too much homework. So far, I do not have much homework today. I am about 95% sure that I will watch television tonight.

17. You are absent from school at least one day during this school year.

18. You have pizza for lunch one day this week.

19. It snows on July 4 this year in Mexico.



20. You get all the problems on your next math test correct.

21. The next baby born in your local hospital is a girl.

22. The sun sets tonight.

23. You win a game by tossing four coins. The result is all heads.

24. You toss a coin and get 100 tails in a row.

Multiple Choice **For Exercises 25–28, choose the fraction closest to the given decimal.**

25. 0.39

A. $\frac{1}{2}$ **B.** $\frac{1}{4}$ **C.** $\frac{1}{8}$ **D.** $\frac{1}{10}$

26. 0.125

F. $\frac{1}{2}$ **G.** $\frac{1}{4}$ **H.** $\frac{1}{8}$ **J.** $\frac{1}{10}$

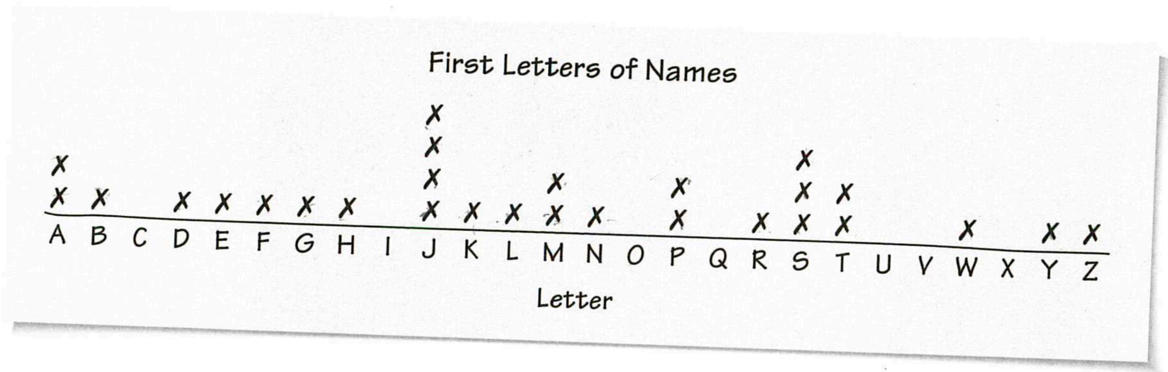
27. 0.195

A. $\frac{1}{2}$ **B.** $\frac{1}{4}$ **C.** $\frac{1}{8}$ **D.** $\frac{1}{10}$

28. 0.24

F. $\frac{1}{2}$ **G.** $\frac{1}{4}$ **H.** $\frac{1}{8}$ **J.** $\frac{1}{10}$

29. Koto's class makes the line plot shown below. Each mark represents the first letter of the name of a student in her class.



Suppose you choose a student at random from Koto's Class.

a. What is the probability that the student's name begins with J?

b. What is the probability that the student's name begins with a letter after F and before T in the alphabet?

c. What is the probability that you choose Koto?

d. Suppose two new students, Melvin and Tara, join the class. You now choose a student at random from the class. What is the probability that the student's name begins with J?

30. A bag contains red, white, blue, and green marbles. The probability of choosing a red marble is $\frac{1}{7}$. The probability of choosing a green marble is $\frac{1}{2}$. The probability of choosing a white marble is half the probability of choosing a red one. You want to find the number of marbles in the bag.

a. Why do you need to know how to multiply and add fractions to proceed?

b. Why do you need to know about multiples of whole numbers to proceed?

c. Can there be seven marbles in the bag? Explain.

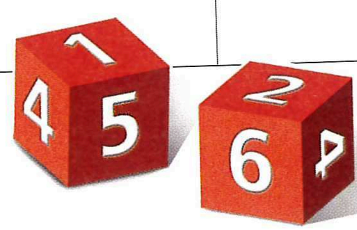
31. Write the following as one fraction.

a. $\frac{1}{2}$ of $\frac{1}{7}$

b. $\frac{1}{7} + \frac{1}{14} + \frac{1}{2}$

32. Karen and Mia play games with coins and number cubes. No matter which game they play, Karen loses more often than Mia. Karen is not sure if she just has bad luck or if the games are unfair. The games are described in this table. Review the game rules and complete the table.

Games	Is It Possible for Karen to Win?	Is It Likely Karen Will Win?	Is the Game Fair or Unfair?
Game 1 Roll a number cube. • Karen scores a point if the roll is even. • Mia scores a point if the roll is odd.			
Game 2 Roll a number cube. • Karen scores a point if the roll is a multiple of 4. • Mia scores a point if the roll is a multiple of 3.			
Game 3 Toss two coins. • Karen scores a point if the coins match. • Mia scores a point if the coins do not match.			
Game 4 Roll two number cubes. • Karen scores a point if the number cubes match. • Mia scores a point if the number cubes do not match.			
Game 5 Roll two number cubes. • Karen scores a point if the product of the two numbers is 7. • Mia scores a point if the sum of the two numbers is 7.			



Connections

33. Karen and Mia invent another game. They roll a number cube twice and read the two digits shown as a two-digit number. So if Karen gets a 6 and then a 2, she has 62.

a. What is the least number possible?

b. What is the greatest number possible?

c. Are all numbers equally likely?

d. Suppose Karen wins on any prime number and Mia wins on any multiple of 4. Explain how to decide who is more likely to win.

Extensions

34. Place 12 objects of the same size and shape in a bag such as blocks or marbles. Use three or four different solid colors.

a. Describe the contents of your bag.

b. Determine the theoretical probability of choosing each color by examining the bag's contents.

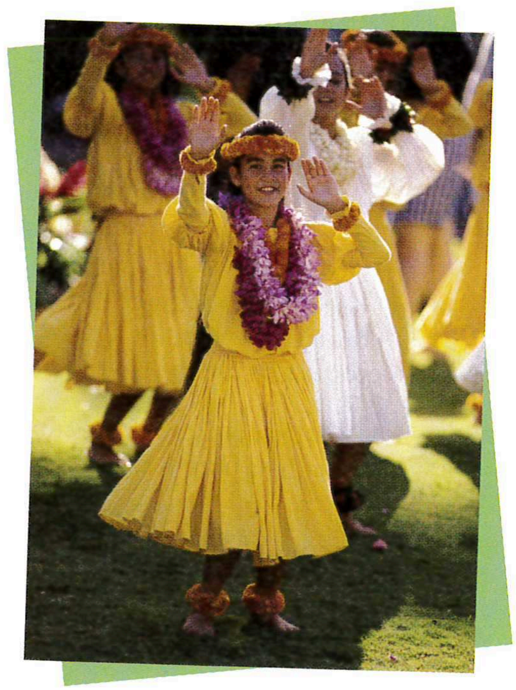
c. Conduct an experiment to determine the experimental probability of choosing each color. Describe your experiment and record your results.

d. How do the two types of probability compare?

35. Suppose you are a contestant on the *Gee Whiz Everyone Wins!* game show in Problem 2.3. You win a mountain bike, a CD player, a vacation to Hawaii, and a one-year membership to an amusement park. You play the bonus round and lose. Then the host makes this offer:

> You can choose from the two bags again. If the two colors match, you win $5,000. If the two colors do not match, you do not get the $5,000 and you return all the prizes.

Would you accept this offer? Explain.

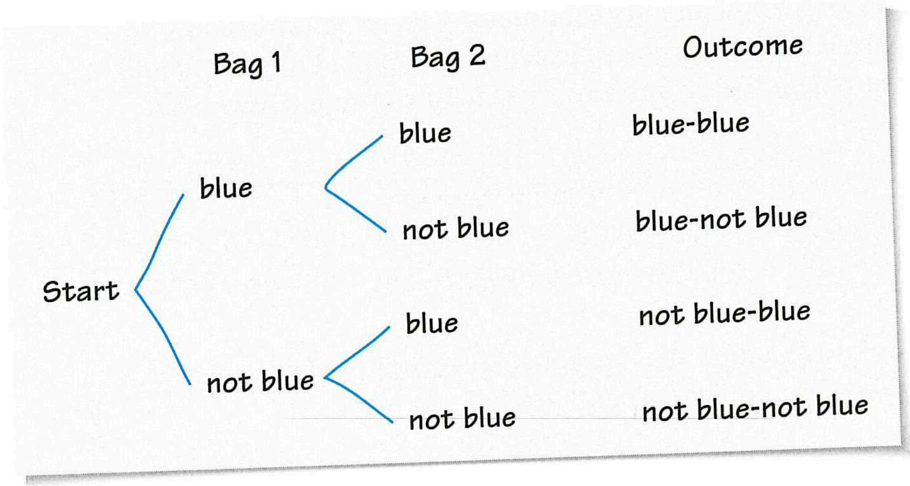


36. Suppose you compete for the bonus prize on the *Gee Whiz Everyone Wins!* game in Problem 2.3. You choose one block from each of two bags. Each bag contains one red, one yellow, and one blue block.

a. Make a tree diagram to show all the possible outcomes.

b. What is the probability that you choose two blocks that are *not* blue?

c. Jason made the tree diagram shown below to find the probability of choosing two blocks that are *not* blue. Using his tree, what probability do you think Jason got?



d. Does your answer in part (b) match Jason's? If not, why do you think Jason gets a different answer?

37. Suppose you toss four coins.

a. List all the possible outcomes.

b. What is the probability of each outcome?

c. Design a game for two players that involves tossing four coins. What is the probability that each player wins? Is one player more likely to win than the other player?

Connections Extensions

Mathematical Reflections 2

In this investigation, you explored two ways to get information about the probability that something will occur. You can design an experiment and collect data (to find experimental probabilities), or you can think about a situation and analyze it carefully to see exactly what might happen (to find theoretical probabilities). These questions will help you summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. Describe how you can find the theoretical probability of an outcome. Why is it called a theoretical probability?

2. a. Suppose two people do an experiment to estimate the probability that an outcome occurs. Will they get the same probabilities? Explain.

   b. Suppose two people analyze a situation to find the theoretical probability that an outcome occurs. Will they get the same probabilities? Explain.

   c. One person uses an experiment to estimate the probability that an outcome occurs. Another person analyzes the situation to find the theoretical probability that the outcome can occur. Will they get the same probabilities? Explain.