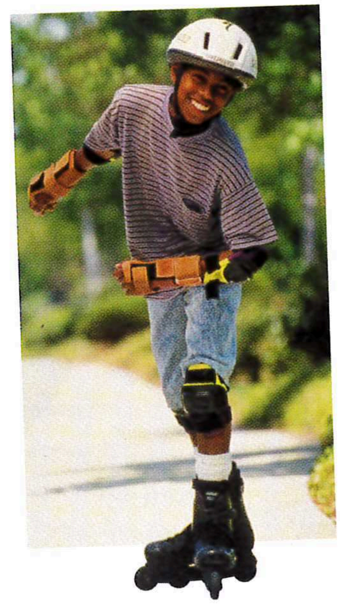
Investigation 3

Making Decisions With Probability

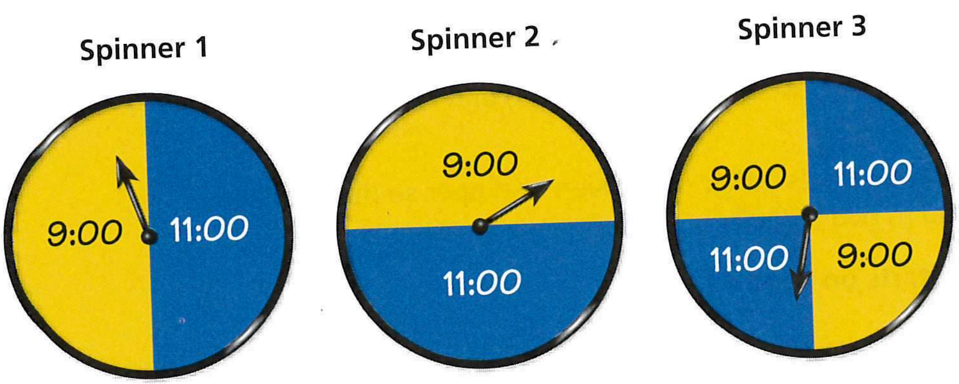


Spring vacation has arrived! Kalvin thinks he can stay up until 11:00 P.M. every night. His father thinks Kalvin will have more energy for his activities (such as roller blading, cleaning out the garage, or washing dishes) during his vacation if he goes to bed at 9:00 P.M.

3.1 Designing a Spinner

Getting Ready for Problem 3.1

Kalvin makes the three spinners shown below. Kalvin hopes that his father lets him use one of the spinners to determine his bedtime.



- Which spinner gives Kalvin the best chance of going to bed at 11:00? Explain.

Kalvin decides to design a spinner that lands on 11:00 the most. To convince his father to use this spinner, Kalvin puts three 9:00 spaces, two 10:00 spaces, and one 11:00 space on the spinner. However, he uses the biggest space for 11:00. Kalvin hopes the pointer lands on that space the most.

Which time do you think is most likely to occur?

Problem 3.1 Finding Probabilities With a Spinner

A. 1. Find the experimental probability that the pointer lands on 9:00, on 10:00, and on 11:00.

2. After how many spins did you decide to stop spinning? Why?

3. Suppose Kalvin spins the pointer 64 times. Based on your experiment, how many times can he expect the pointer to land on 9:00, on 10:00, and on 11:00?

B. 1. What is the theoretical probability that the pointer lands on 9:00, on 10:00, and on 11:00? Explain.

2. Suppose Kalvin spins the pointer 64 times. Based on your theoretical probabilities, how many times can he expect the pointer to land on 9:00, on 10:00, and on 11:00?

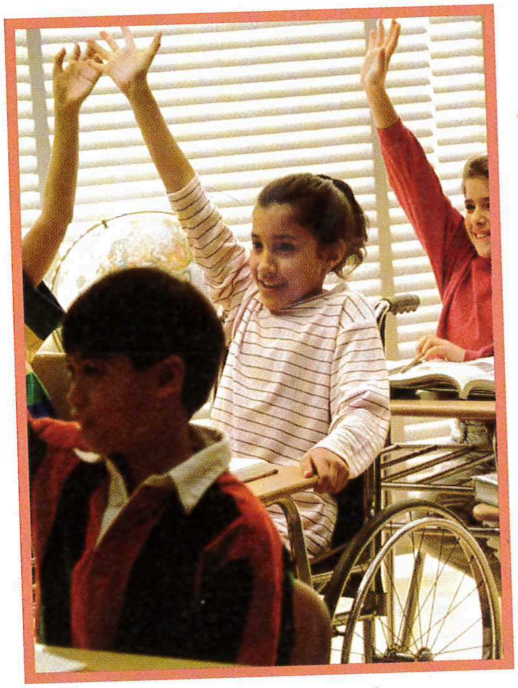
3. How do your answers to Question A part (3) and Question B part (2) compare?

C. Describe one way Kalvin's father can design a spinner so that Kalvin is most likely to go to bed at 9:00.

ACE **Homework starts on page 44.**

3.2 Making Decisions

Kalvin begins to think that probability is a good way to make decisions. One day at school, Kalvin's teacher, Ms. Miller, has to decide which student to send to the office to get an important message. Billie, Evo, and Carla volunteer. Kalvin suggests they design a quick experiment to choose the student fairly.



Getting Ready for Problem 3.2

Which of these items can Kalvin's class use to choose a messenger? How can they make the decision fair?

- a coin
- a six-sided number cube
- colored cubes
- playing cards
- a spinner

Problem 3.2 Analyzing Fairness

Two suggestions for making a decision are shown in each question. Decide whether the suggestions are fair ways to make the decision. Explain your reasoning.

A. At lunch, Kalvin and his friends discuss whether to play kickball, soccer, baseball, or dodgeball. Ethan and Ava each have a suggestion.

Ethan: We can make a spinner that looks like this:

Ava: We can roll a number cube. If it lands on 1, we play kickball. A roll of 2 means soccer, 3 means baseball, 4 means dodgeball, and we can roll again if it's 5 or 6.

B. The group decides to play baseball. Tony and Meda are the team captains. Now they must decide who bats first.

Tony: We can roll a number cube. If the number is a multiple of three, my team bats first. Otherwise, Meda's team bats first.

Meda: Yes, let's roll a number cube, but my team bats first if the number is even and Tony's team bats first if it's odd.

C. There are 60 sixth-grade students at Kalvin's school. The students need to choose someone to wear the mascot costume on field day.

Huey: We can give everyone a number from 1 to 60. Then, we can roll 10 number cubes and add the results. The person whose number is equal to the sum wears the costume.

Sal: That doesn't seem fair. Everyone should have a number from 0 to 59. In one bag, we can have blocks numbered 0 to 5. In another bag, we can have blocks numbered 0 to 9. We can select one block from the first bag to represent the tens digit and one block from the second bag to represent the ones digit.

ACE Homework starts on page 44.

3.3 Scratching Spots

Have you ever tried to win a contest? Probability can often help you figure out your chances of winning.

Tawanda's Toys is having a contest. Any customer who spends at least $10 receives a scratch-off prize card.

- Each card has five gold spots that reveal the names of video games when you scratch them.
- Exactly two spots match on each card.
- A customer may scratch off only two spots on a card.
- If the spots match, the customer wins that video game.

It can be difficult to get enough prize cards to conduct an experiment. So, you can design a related experiment to help you find the probability of each outcome. A model used to find experimental probabilities is a **simulation.**

One way you can simulate the scratch-off card is by using five playing cards. First, make sure that exactly two out of the five cards match. Place the cards facedown on a table. While your eyes are closed, have a friend mix up the cards. Then open your eyes and choose two cards. If the cards match, you win. Otherwise, you lose.

Can you think of another way to simulate the scratch-off cards?

Problem 3.3 Using a Simulation

A. Use the card simulation above to find the probability of winning.

B. Examine the different ways you can scratch off two spots. Find the theoretical probability of winning with one prize card.

C. Suppose you have 100 prize cards from Tawanda.

1. How many video games can you expect to win?

2. How much money do you need to get 100 cards?

D. Tawanda thinks she may lose money with this promotion. The video games she gives away cost her $15 each. Will Tawanda lose money? Why or why not?

ACE **Homework starts on page 44.**

Applications Connections Extensions

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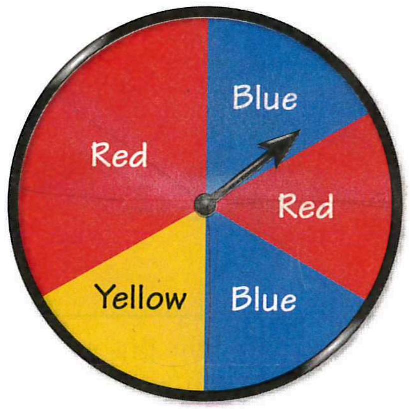
Applications

1. For parts (a)–(g), use a spinner similar to the one below.



a. Use a paper clip or bobby pin as a pointer. Spin the pointer 30 times. What fraction of your spins land on red? What fraction land on blue? On yellow?

b. Use an angle ruler or another method to examine the spinner. What fraction of the spinner is red? What fraction is blue? What fraction is yellow? Explain.

c. Compare your answers to parts (a) and (b). Do you expect these answers to be the same? Why or why not?

d. Suppose you spin 300 times instead of 30 times. Do you expect your answers to become closer to or further from the fractions you found in part (b)? Explain your reasoning.

e. When you spin, is it equally likely that the pointer will land on red, on blue, or on yellow? Explain.

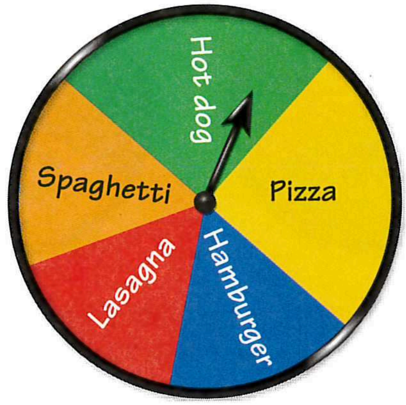
f. Suppose you use the spinner to play a game with a friend. Your friend scores a point every time the pointer lands on red. To make the game fair, for what outcomes should you score a point? Explain.

g. Suppose you use this spinner to play a three-person game. Player A scores if the pointer lands on yellow. Player B scores if the pointer lands on red. Player C scores if the pointer lands on blue. How can you assign points so that the game is fair?

2. The cooks at Kyla's school make the spinners below to help them choose the lunch menu. They let the students take turns spinning. For parts (a)–(c), decide which spinner you would choose. Explain your reasoning.

Spinner A

Spinner B



a. Your favorite lunch is pizza.

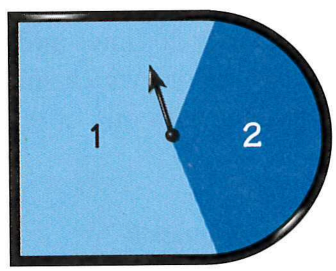
b. Your favorite lunch is lasagna.

c. Your favorite lunch is hot dogs.

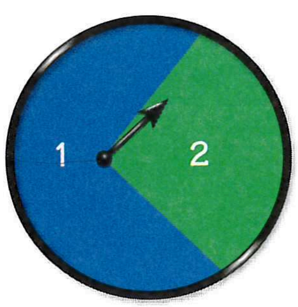
3. When you use each of the spinners below, the two possible outcomes are landing on 1 and landing on 2. Are the outcomes equally likely? If not, which outcome has a greater theoretical probability? Explain.

a.

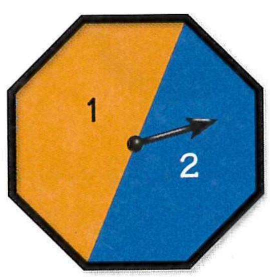
b.



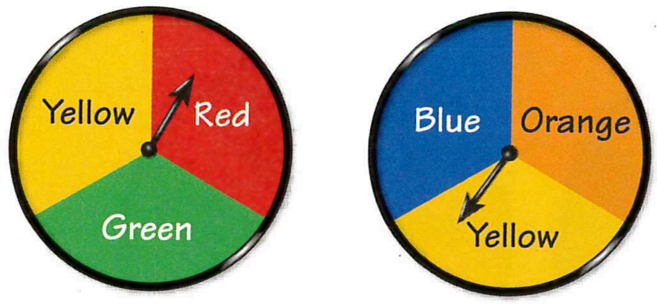
c.



d.



4. A science club hosts a carnival to raise money. A game called Making Purple at the carnival involves using both of the spinners shown. If the player gets red on spinner A and blue on spinner B, the player wins because mixing red and blue makes purple.



a. List the outcomes that are possible when you spin both pointers. Are these outcomes equally likely? Explain your reasoning.

b. What is the theoretical probability that a player "makes purple"? Explain.

c. If 100 people play the Making Purple game, how many people do you expect to win?

d. The club charges $1 per turn. A player who makes purple wins $5. Suppose 100 people play. How much money do you expect the club to make?

5. Molly designs a game for a class project. She makes the three spinners shown. She tests to see which one she likes best for her game. She spins each pointer 20 times and writes down her results, but she forgets to record which spinner gives which set of data. Match each spinner with one of the data sets. Explain your answer.

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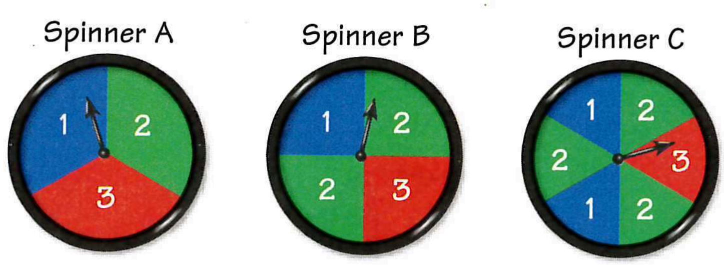

First data set:	1, 2, 3, 2, 1, 1, 2, 1, 2, 2, 2, 3, 2, 1, 2, 2, 2, 3, 2, 2
Second data set:	2, 3, 1, 1, 3, 3, 3, 1, 1, 2, 3, 2, 2, 2, 1, 1, 1, 3, 3, 3
Third data set:	1, 2, 3, 3, 1, 2, 2, 2, 3, 2, 1, 2, 2, 2, 3, 2, 2, 3, 2, 1

6. Three people play a game on each spinner in Exercise 5. Player 1 scores a point if the pointer lands on 1. Player 2 scores a point if the pointer lands on 2. Player 3 scores a point if the pointer lands on 3.

a. On which spinner(s) is the game a fair game? Why?

b. Choose a spinner that you think doesn't make a fair game. Then, change the scoring rules to make the game fair by assigning different points for landing on the different numbers. Explain why your point system works.

7. a. Make a spinner and a set of rules for a fair two-person game. Explain why your game is fair.

b. Make a spinner and a set of rules for a two-person game that is *not* fair. Explain why your game is not fair.

8. **Multiple Choice** Jake, Carl, and John try to decide what to do after school. Jake thinks they should play video games. Carl wants to see a movie. John thinks they should ride their bikes. Which choice is a fair way to decide?

A. Let's toss three coins. If they all match, we play video games. If there are exactly two heads, we see a movie. If there are exactly two tails, we ride our bikes.

B. Let's roll a number cube. If we roll a 1 or 2, we play video games. If we roll a 3 or 4, we go to the movies. Otherwise, we ride bikes.

C. Let's use this spinner.

D. None of these is fair.

9. **Multiple Choice** The Millers can't decide whether to eat pizza or burritos for dinner.

F. Let's roll a number cube and toss a coin. If the number cube is even and the coin is heads, then we eat pizza. If the number cube is odd and the coin is tails, then we eat burritos. If neither happens, we try again.

G. Let's toss a coin. If it is heads, we eat pizza. If it is tails, we do *not* eat burritos.

H. Each of these is fair.

J. Neither of these is fair.

10. Tawanda wants fewer winners for her scratch-off cards. She orders new cards with six spots. Two of the spots on each card match. What is the probability that a person who plays once will win on the card?

Connections

For Exercises 11–16, complete the following table. Write each probability as a fraction, decimal, or percent.

Probabilities

	Fraction	Decimal	Percent
11.	$\frac{1}{4}$	■	25%
12.	$\frac{1}{8}$	■	■
13.	■	■	$33\frac{1}{3}$%
14.	■	■	10%
15.	■	0.1666...	■
16.	■	0.05	■

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17. The cooks at Kyla's school let students make spinners to determine the lunch menu.

a. Make a spinner for which the chance of lasagna is 25%, the chance of a hamburger is $16\frac{2}{3}$% and the chance of a tuna sandwich is $33\frac{1}{3}$%. The last choice is hot dogs.

b. What is the chance of hot dogs?

18. Three of the following situations have the same probability of getting "spinach." What is the probability for these three situations?

a. Spin the pointer on this spinner once.

b. Roll a number cube once. You get "spinach" when you roll a multiple of 3.

c. Toss two coins. You get "spinach" with one head and one tail.

d. Roll a number cube once. You get "spinach" when you roll a 5 or 6.

For Exercises 19–21, rewrite each pair of numbers. Insert <, >, or = to make a true statement.

19. $\frac{1}{3\frac{1}{2}}$ ■ $\frac{1}{4}$ **20.** $\frac{3.5}{7}$ ■ $\frac{1}{2}$ **21.** 0.30 ■ $\frac{1}{3}$

22. Use the table of historic baseball statistics to answer parts (a)–(d).

Batting Averages

Player	At Bats	Hits
Nomar Garciaparra	4,089	1,317
Derek Jeter	5,457	1,715
Jackie Robinson	4,877	1,518

a. What percent of Nomar Garciaparra's at bats resulted in a hit?

b. What percent of Derek Jeter's at bats resulted in a hit?

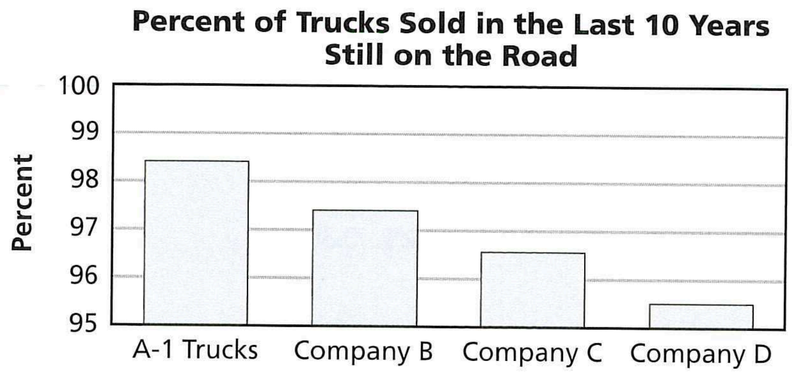
c. What percent of Jackie Robinson's at bats resulted in a hit?

d. Suppose each player comes to bat today with the same skill his record shows. Who has the greatest chance of getting a hit? Explain.

For Exercises 23–25, rewrite each fraction as an equivalent fraction using a denominator of 10 or 100. Then, write a decimal number for each fraction.

23. $\frac{3}{20}$ **24.** $\frac{2}{5}$ **25.** $\frac{11}{25}$

26. A-1 Trucks used this graph to show that their trucks last longer than other trucks.



a. The bar for A-1 Trucks is about six times the height of Company D's bar. Does this mean that the chance of one of A-1's trucks lasting ten years is about six times as great as the chance of one of Company D's trucks lasting ten years? Explain.

b. If you wanted to buy a truck, would this graph convince you to buy a truck from A-1 Trucks? Why or why not?

27. The Federal Trade Commission (FTC) makes rules for businesses that buy and sell things. One rule states that an advertisement may be found unlawful if it can deceive a person.

To decide whether an ad is deceptive, the FTC considers the "general impression" it makes on a "reasonable person." Even if every statement is true, the ad is deceptive if it gives an overall false impression. For example, cows can't appear in margarine ads because it gives the false impression that margarine is a dairy product.

a. Tawanda places this ad in a newspaper. Qualifying customers receive a prize card like the ones described in the introduction to Problem 3.3. According to the FTC, is it legal for Tawanda to say, "Every card is a winner"? Explain.

b. Design a better ad that excites people but does not lead some to think they will win every time.

c. Find an ad that might be deceptive. Why do you think it is deceptive? What proof could the company provide to change your mind?

28. A sugarless gum company used to have an advertisement that stated:

Four out of five dentists surveyed recommend sugarless gum for their patients who chew gum.

Do you think this statement means that 80% of dentists believe their patients should chew sugarless gum? Explain your reasoning.

29. Portland Middle School students make a flag as shown. After it hangs outside for a month, it looks dirty so they examine it. They find more bugs stuck on the yellow part than on the green part. Cheng says bugs are more attracted to yellow than to green.

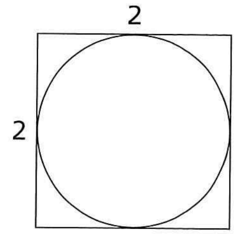
a. Students in a science class test Cheng's conjecture with a design the same as the flag design. Suppose Cheng's conjecture is true. What is the chance that a bug landing at random on the flag hits the yellow part?

b. Suppose 13 bugs land on the yellow part and 12 bugs land on the green part. Is this evidence that supports Cheng's conjecture?

Connections

Did You Know?

Pi can be estimated using probability. Take a square that is 2 units on each side and inscribe a circle inside which has a radius of 1 unit.



The area of the square is 4 square units and the area of the circle is $\pi \cdot r^2$ or $\pi \cdot 1^2 = \pi$ square units.

The ratio of the area of the circle to the area of the square is $\frac{\pi}{4}$. Ratios can be written as fractions.

Computer simulations can be done where the computer randomly places a dot inside the square. A computer can place 10,000 dots inside the square in less than 30 seconds.

This ratio $\frac{\text{number of dots inside the circle}}{\text{total number of dots inside the square}}$ should approximate $\frac{\pi}{4}$.

So, π should equal four times the ratio $\frac{\text{number of dots inside the circle}}{\text{total number of dots inside the square}}$.

30. Charlie runs three computer simulations such as the one described in the Did You Know? He records data for the three trials.

a. Complete the table below of Charlie's data.

Pi Estimations

Trial	Dots Inside the Circle	Dots Inside the Square	Ratio: $\frac{\text{Dots in Circle}}{\text{Dots in Square}}$
1	388	500	■
2	352	450	■
3	373	475	■

b. Decide which trial is closest to an approximation for $\frac{\pi}{4}$. Explain your reasoning.

Extensions

31. Design a spinner with five regions so that the chances of landing in each region are equally likely. Give the number of degrees in the central angle of each region.

32. Design a spinner with five regions so that the chances of landing in one region are twice the chances of landing in each of the other four regions. Give the number of degrees in the central angle of each region.

For Exercises 33–35, design a contest for each company. Each contest should help the company attract customers, but not make the company lose money. Explain the rules, including any requirements for entering the contest.

33. The manager of a small clothing store wants to design a contest in which 1 of every 30 players wins a prize.

34. The director of operations for a chain of supermarkets wants to design a contest with a $100,000 grand prize!

35. An auto store sells new and used cars. The owner wants to have a contest with lots of winners and big prizes. She wants about one of every ten players to win a $500 prize.

Mathematical Reflections 3

In this investigation, you used spinners and cubes in probability situations. You used both experimental and theoretical probabilities to help you make decisions. These questions will help you summarize what you learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. Describe a situation in which you and a friend can use probability to make a decision. Can the probabilities of the outcomes be determined both experimentally and theoretically? Why or why not?
2. Describe a situation in which it is difficult or impossible to find the theoretical probabilities of the outcomes.
3. Explain what it means for a probability situation to be fair.