Investigation 4

Probability, Genetics, and Games

Have you ever heard of genes? (We don't mean the kind you wear!) What color are your eyes? Can you curl your tongue? Your birth parents gave you a unique set of genes that determine such things.

Scientists who study traits such as eye and hair color are called geneticists (juh NET uh sists). Geneticists use probability to predict certain traits in children based on traits in their parents or relatives.

4.1 Genetic Traits

Look at the earlobe of a classmate. Is it attached or does it dangle freely? The type of earlobe you have is a trait determined by your genes. Here is a description of four genetic traits:

- *Attached earlobe*: An earlobe is attached if its lowest point is attached directly to the head, as shown below.
- *Dimple*: A dimple is a small indentation, usually near the mouth.
- *Straight hair*: Straight hair has no waves or curls. (Note: Consider only how a person's hair is naturally.)
- *Widow's peak*: A widow's peak is a V-shaped hairline, as shown below.

Attached earlobe

Unattached earlobe

Widow's peak

No widow's peak

Problem 4.1 Applying Experimental Probability

The table lists four genetic traits.

Classroom Genetics Survey

Trait	Yes	No	Total
Attached Earlobes			
Dimples			
Straight Hair			
Widow's Peak			

A. Copy the table. Find the number of people in your class who have each trait and record the results in your table.

B. Use your table to complete parts (1)–(4).

1. For each trait, find the probability that a person chosen at random has the trait.
2. What is the probability that a person chosen at random does *not* have straight hair?
3. How many students in your school do you expect to have attached earlobes?
4. How many students in your school do you expect to have a widow's peak?

C. Below are the results of a study of students from around the country.

U.S. Genetics Survey

Trait	Yes	No
Attached Earlobes	443	1,080
Dimples	445	1,066
Straight Hair	623	666
Widow's Peak	734	777

1. Find the probability that a person chosen at random has each trait.
2. How do the probabilities in Question B compare to the probabilities from the national data?

ACE Homework starts on page 62.

4.2 Tracing Traits

In the last problem, you looked at experimental probabilities for certain traits. In some cases, you can determine the probability that a child will have a trait based on his or her parents' genes.

Geneticists use the word *allele* (uh LEEL) for one of a pair of genes that determines a trait. For example, you have two alleles that determine whether your earlobes are attached. You receive one of these alleles from your birth mother and one from your birth father. Of course, each parent has two earlobe alleles.

Let's use e to represent the allele for attached earlobes. Let E represent the allele for nonattached earlobes. If you receive an e allele from each parent, your earlobe alleles will be ee, and you will have attached earlobes. If you receive an E allele from each parent, your earlobe alleles will be EE. Then you will have nonattached earlobes.

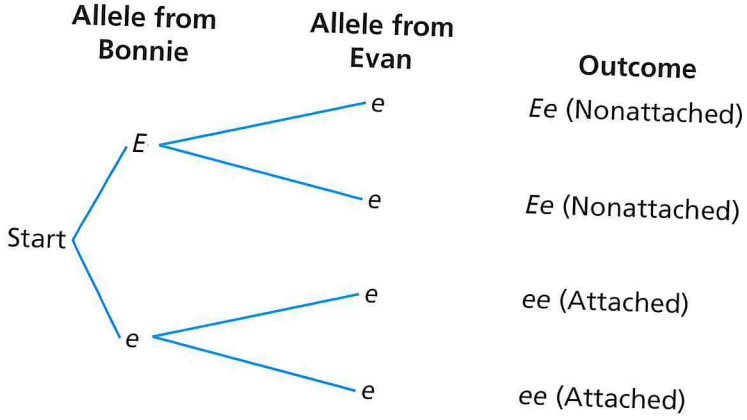
What if you receive one E and one e allele? In nature, the E allele is *dominant* and the e allele is *recessive*. This means that you have an Ee combination, the E dominates, and you will have nonattached earlobes.

Earlobe Alleles

Letters	Earlobe Trait
EE	Nonattached
Ee or eE	Nonattached
ee	Attached

An Example: Bonnie and Evan's Baby

Bonnie and Evan are going to have a baby. Bonnie's earlobe alleles are *Ee*, and Evan's earlobe alleles are *ee*. You can determine the probability that their baby will have attached earlobes by making a tree diagram.



There are four possible allele pairs (outcomes). Two of these pairs, *ee* and *ee*, result in attached earlobes. The probability that Bonnie and Evan's baby will have attached earlobes is $\frac{2}{4}$, or $\frac{1}{2}$.

You can also find the probabilities by making a table such as the one at the right. List Evan's alleles along the side and Bonnie's alleles on top. The four white squares show the possible combinations.

		Bonnie	
		E	***e***
Evan	***e***	*Ee*	*ee*
	e	*Ee*	*ee*

Did You Know?

Bonnie and Evan's chart is sometimes called a *Punnett square* by geneticists. A Punnett square is a chart which predicts all possible gene combinations. Punnett squares are named for an English geneticist, Reginald Punnett. He discovered some basic principles of genetics. He studied the feather color traits of chickens in order to quickly determine whether chickens were male or female when they were born.

Go Online PHSchool.com **For:** Information about Punnett squares **Web Code:** ame-9031

Problem 4.2 Applying Theoretical Probability

In Questions A–C, examine each family situation and answer the questions.

A. Dasan's mother is expecting her third child. His mother and father both have the earlobe alleles *Ee*.

1. What is the probability that Dasan's new sibling will have attached earlobes?
2. What is the probability that his new sibling will have nonattached earlobes?

B. Geoff's earlobe alleles are *EE* and Mali's earlobe alleles are *Ee*. What is the probability that their child will have nonattached earlobes?

C. Both of Eileen's parents have attached earlobes. What is the probability that Eileen has attached earlobes?

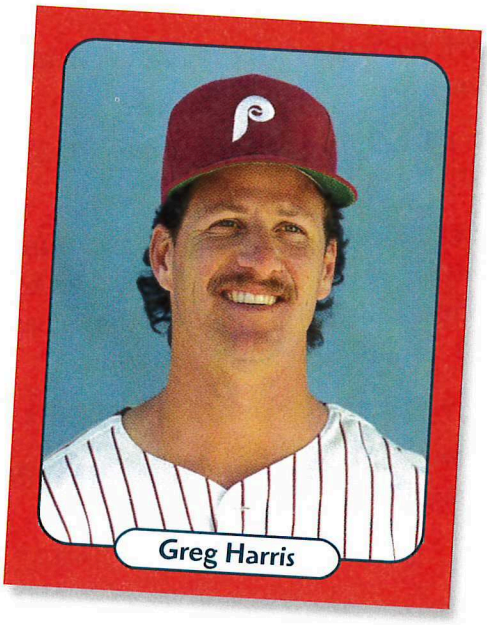
ACE **Homework starts on page 62.**

There are many other traits you can study in the way you studied earlobes. For example, having certain characteristics (dimples, curly or wavy hair, and widow's peak) is dominant over not having the characteristics.

Did You Know?

There are dominant traits that do not show up very often in the population. For example, the trait for six fingers on one hand is a dominant trait, and five fingers is a recessive trait. Because only a few people carry the allele for six fingers, very few people are born with this trait.

Greg Harris, a major-league baseball player from 1981 to 1995, has six fingers on one hand. He had a specially designed, reversible six-fingered glove. In 1995, he became the only pitcher since 1900 to pitch with both hands in a major-league game.


Greg Harris

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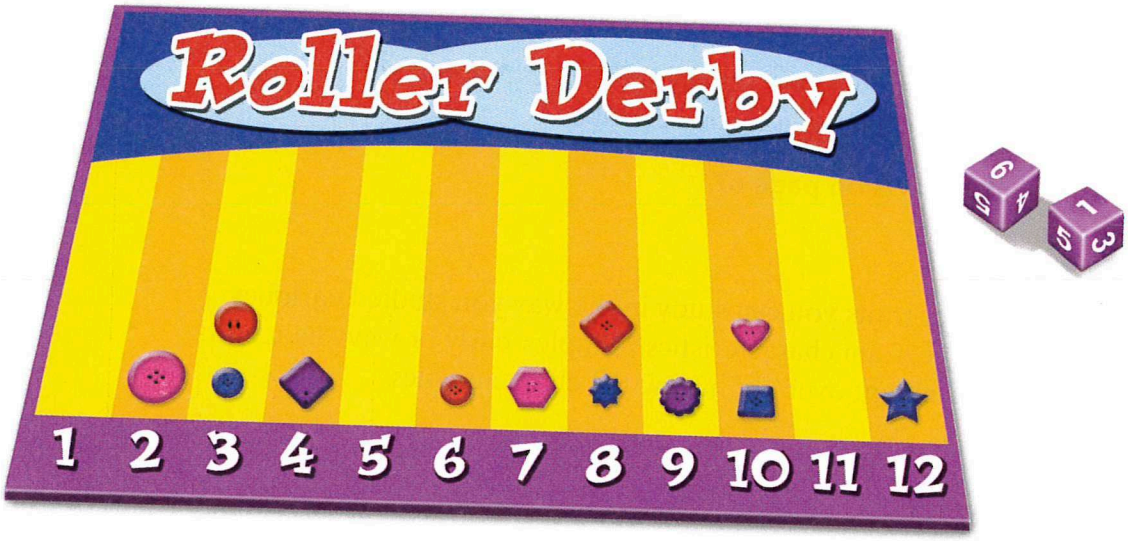
4.3 Roller Derby

Have you ever figured out a strategy for winning a game?

Now that you know about making tables and diagrams to find probabilities, you can use these tools to find winning strategies for games. In this problem, you play a two-team game called Roller Derby.

Each team needs a game board with columns numbered 1–12, a pair of number cubes, and 12 markers (such as coins, buttons, or small blocks).



Roller Derby Rules

1. Each team places its 12 markers into their columns in any way it chooses.
2. Each team rolls a number cube. The team with the highest roll goes first.
3. Teams take turns rolling the two number cubes. They remove a marker from the column on their board with the same number as the total sum of the numbers on the number cubes. If the column is empty, the team does not get to remove a marker.
4. The first team to remove all the markers from its board wins.

As you play, think about strategies for winning and how probability relates to your strategies.

Problem 4.3 Analyzing a Game

A. Play the game at least twice. For each game, record the strategies you use to place your markers on the board. Also, record how many times each sum is rolled. What is a good strategy for placing your markers on the game board?

B. **1.** Which sums seem to occur most often?

2. Which sums do not come up very often?

C. Find all the possible outcomes (number pairs) of rolling two number cubes. Find the sums for each of these outcomes.

1. Are all the sums equally likely? Explain.

2. How many ways can you get a sum of 2?

3. What is the probability of getting a sum of 4?

4. What is the probability of getting a sum of 6?

5. Which sums occur most often?

D. Now that you have looked at the possible outcomes of the Roller Derby game, do you have any new strategies for winning? Explain.

ACE Homework starts on page 62.

Did You Know?

Galileo was an Italian physicist, astronomer, and mathematician. He is famous for helping develop a model in which the sun was the center of the universe. He also studied problems in probability similar to the ones you have seen.

A famous problem he worked on involved rolling three number cubes. He looked at the possibilities for getting a sum of 9 or a sum of 10. A sum of 9 is made using six groups of numbers:

(1, 2, 6), (1, 3, 5), (1, 4, 4), (2, 2, 5), (2, 3, 4), and (3, 3, 3).

A sum of 10 is made using six other groups of numbers:

(1, 3, 6), (1, 4, 5), (2, 2, 6), (2, 3, 5), (2, 4, 4), and (3, 3, 4).

What puzzled people is that, when they did experiments, the sum of 10 occurred more often. By making a diagram similar to a counting tree, Galileo showed the theoretical probability matched the experimental results. There are actually 25 combinations that have a sum of 9 and 27 combinations that have a sum of 10.

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For: Information about Galileo
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Applications Connections Extensions

ACE

Applications

1. A foot arch is a genetic trait. A foot arch is a space between the middle of a person's foot and the floor when the person stands. In a national study, 982 people said they had a foot arch, while 445 people said they did not have a foot arch.

   a. Based on these data, what is the experimental probability that a person chosen at random has a foot arch?

   b. In a recent year, about 16,600 people participated in the Boston Marathon. Use the data above to estimate the number of participants who did *not* have a foot arch. Explain.

   c. If you know people who are runners, find out if they have foot arches. Does your data seem to match the national study data?

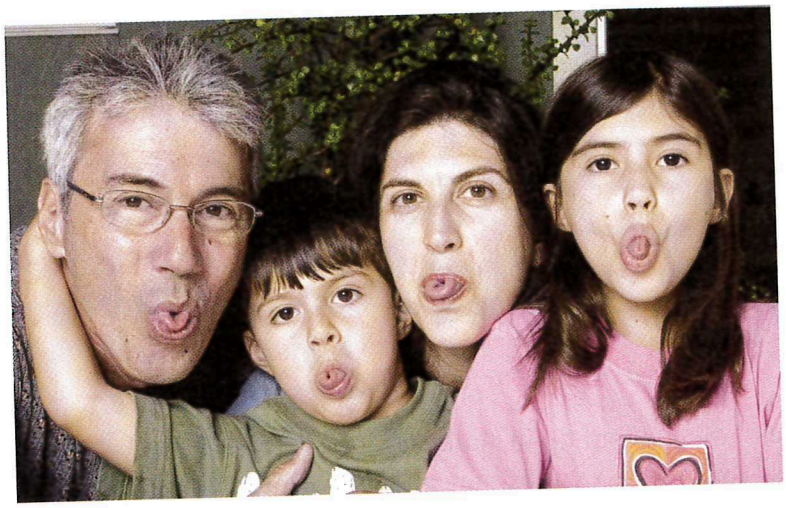
2. Some genetic traits are gender-linked. These traits are more prevalent in people of one gender than the other. For example, color blindness is far more common in men than in women. About 7% of the U.S. male population either cannot distinguish red from green, or sees red and green differently from most people. Red-green color blindness only affects about 0.4% of U.S. females.

   About 550 males and 600 females attend a middle school. How many males and females do you predict have red-green color blindness?

For Exercises 3–7, use the following information about the genetics of tongue curling to answer the question.

Let T stand for the allele for tongue curling and let t stand for the allele for non-curling. T is dominant, so people with TT or Tt can curl their tongues, while people with tt cannot.

3. Neither Greg nor Megan can curl their tongues. What is the probability that their daughter can curl her tongue? Explain.



4. Suppose a woman with tongue-curling alleles TT and her husband with tongue-curling alleles tt are expecting a baby. What is the probability that the baby will be able to curl his tongue? Explain.

5. If Laura can curl her tongue, is it possible that neither of her parents can curl their tongues? Why or why not?

6. Suppose Ryan can't curl his tongue. Is it possible that both of his parents can curl their tongues? Why or why not?

7. Suppose both Niran and Gen can curl their tongues. They are wondering how many of their children will have this ability.

a. Gen's mother can curl her tongue, but her father can't. What are Gen's tongue-curling alleles? Explain.

b. Niran's mother can't curl her tongue, but his father can. What are Niran's tongue-curling alleles? Explain.

c. What is the probability that Niran and Gen's first child will have the tongue-curling ability?

d. Suppose their first child has the tongue-curling ability. What is the probability that their second child will also have this ability?

e. Suppose Niran and Gen have ten children. How many of their children would you expect to have the tongue-curling ability? Why?

Multiple Choice **For Exercises 8 and 9, use your list of possible outcomes when you roll two number cubes from Problem 4.3.**

8. What is the probability of getting a sum of 5 when you roll two number cubes?

A. $\frac{1}{9}$ **B.** $\frac{1}{6}$ **C.** $\frac{1}{4}$ **D.** $\frac{1}{3}$

9. What is the probability of getting a sum greater than 9 when you roll two number cubes?

F. $\frac{1}{9}$ **G.** $\frac{1}{6}$ **H.** $\frac{1}{4}$ **J.** $\frac{1}{3}$

Multiple Choice **For Exercises 10 and 11, Ella is playing Roller Derby with Carlos. Ella places all her markers in column 1 and Carlos places all of his markers in column 12.**

10. What is the probability that Ella will win?

A. 0 **B.** $\frac{1}{3}$ **C.** $\frac{1}{2}$ **D.** 1

11. What is the probability that Carlos will win?

F. 0 **G.** $\frac{1}{3}$ **H.** $\frac{1}{2}$ **J.** 1

12. In some board games, you can end up in "jail." One way to get out of jail is to roll doubles (two number cubes that match). What is the probability of getting out of jail on your turn by rolling doubles? Use your list of possible outcomes of rolling two number cubes. Explain your reasoning.

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Connections

For Exercises 13–17, use the data below to answer the question. If there is not enough information to answer the question, explain what additional information you need.

Careers and Goals for Young People

Career/Goal	Ages 8–12	Ages 13–21
Be a millionaire	65%	65%
Star in movie/TV	34%	47%
Make movies	32%	43%
Be a famous athlete	32%	24%
Be a musician/singer	28%	46%
Cure a disease	27%	43%
Start a big company	22%	37%
Be President of the United States	22%	18%
Be a famous writer	22%	33%
Win a Nobel Prize	20%	28%

SOURCE: *Harris Interactive Youthpulse*

13. Which group is more likely to want to be a musician/singer?

14. In a group of 1,500 young people (ages 13–21), about how many would choose to cure a disease?

15. Order the five lowest career/goal choices from least to greatest for ages 13–21.

16. About how many people in your school would select famous athlete as a career?

17. In order to find the percent of all young people (ages 8–12) who want to star in a movie/TV or make a movie, can you add the percents for the two careers/goals together? Why or why not?

18. Suppose you try to determine Fia's and Tomas's earlobe alleles. Here is the information you have:

- Fia has attached earlobes.
- Tomas has nonattached earlobes.
- Their two daughters have nonattached earlobes.
- Their son has attached earlobes.

a. What are Fia's earlobe alleles?

b. What are Tomas's earlobe alleles?

c. If they have another child, what is the probability that the child will have attached earlobes?

19. In *Shapes and Designs*, you built triangles and parallelograms with a given set of criteria. You know that sometimes two people can construct different geometric shapes, given the same set of directions.

a. Suppose your teacher tells you the lengths of all three sides of a given triangle. What is the probability that you construct a triangle congruent to the one that your teacher has in mind? Explain.

b. Suppose your teacher tells you the lengths of all four sides of a given parallelogram. What is the probability that you construct a parallelogram congruent to the parallelogram that your teacher has in mind? Explain.

c. Suppose your teacher tells you the lengths of all four sides of a given rectangle. What is the probability that you construct a rectangle congruent to the one that your teacher has in mind? Explain.

d. Suppose your teacher tells you the perimeter of a given rectangle. What is the probability that you construct a rectangle congruent to the one that your teacher has in mind?

e. Suppose your teacher tells you the lengths of all four sides and the area of a given parallelogram. What is the probability that you construct a parallelogram congruent to the one that your teacher has in mind?

For Exercises 20–23, use your list of possible outcomes when you roll two number cubes to help you answer the questions.

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20. What is the probability that the sum is a multiple of 4?

21. What is the probability that the sum is a common multiple of 2 and 3?

22. What is the probability that the sum is a prime number? Explain.

23. Which has a greater probability of being rolled on a pair of number cubes, a sum that is a factor of 6 or a sum that is a multiple of 6? Explain.

24. Suppose Jose and Nina play the game Evens and Odds. To play the game, they roll two number cubes and find the product of the numbers. If the product is odd, Nina scores a point. If the product is even, Jose scores a point.

a. Make a table of the possible products of two number cubes.

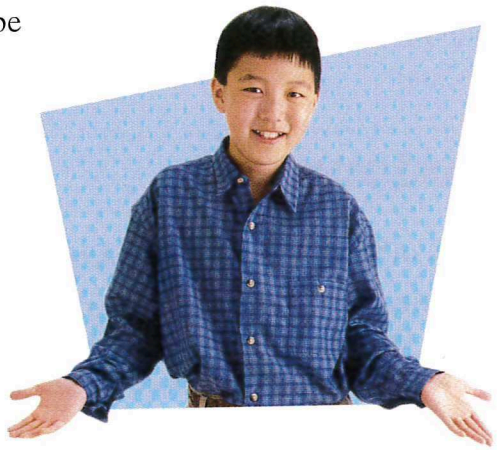
b. What is the probability that Nina wins? What is the probability that Jose wins? Explain your reasoning.

c. Is this a fair game? If not, how could you change the points scored by each player so that it would be fair?

d. What is the probability that the product is a prime number?

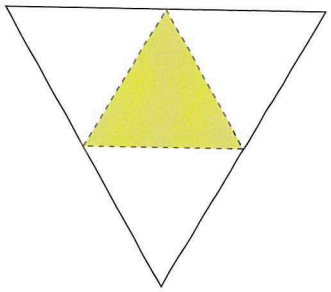
e. What is the probability that the product is a factor of 4?

25. Aran knows that if you roll a number cube once, there is a 50% chance of getting an even number. He says that if you roll a number cube twice, the chance of getting at least one even number is doubled. Is he correct?



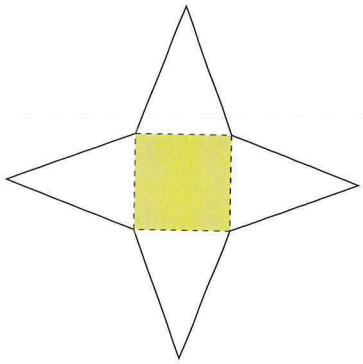
Connections

26. **a.** Suppose you fold this shape along the dashed lines to make a three-dimensional shape. How many faces will it have?



b. Suppose you roll the shape in part (a). What is the probability that the shaded face lands on the bottom?

c. Suppose you fold the shape below. Can you use it in a game? Will the game be fair? Explain.



Extensions

27. Pick one of the following two options:

a. Investigate the earlobes in your family. Make a family tree that shows the earlobe alleles that you can find for each person. Trace back as many generations as you can.

b. Survey a large number of people to estimate the percentage of people in the population who have attached earlobes. Represent the data in a graph.

Mathematical Reflections 4

In this investigation, you explored probability related to genetics and games. You collected data to determine the probability that a person selected at random would have a genetic trait. You also found the probability that a child would have a trait based on information about his or her parents' genes. You examined a game to determine winning strategies for playing the game. These questions will help you summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. How can you collect the data to find the experimental probability that a person chosen at random from your school has a particular trait?
2. Describe a way you can find the theoretical probability that a baby will have a trait, such as attached earlobes or tongue-curling ability. Base your answer on the alleles of the parents.
3. Describe some of the strategies for determining the theoretical probabilities for situations in this unit. Give an example of a situation for each of the strategies.