Investigation 3

Rules and Equations

In the last investigation, you used tables and graphs of relationships to find values of one variable for given values of the other variable. In some cases, you could only estimate or predict a value.

For some relationships, you can write an equation, or formula, to show how the variables are related. Using an equation is often the most accurate way to find values of a variable.

In this investigation, you will use the patterns in tables to help you write equations for relationships. You will then use your equations to compute values of the dependent variable for specific values of the independent variable.

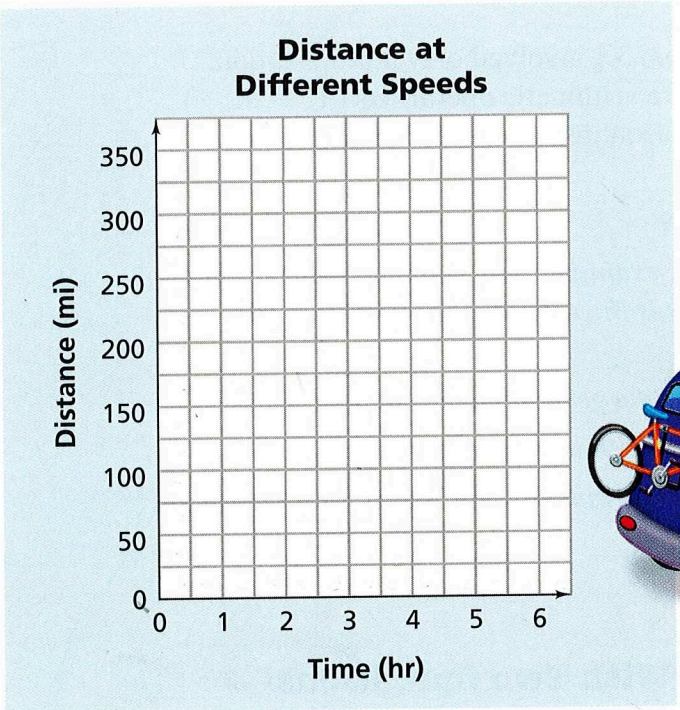
3.1 Writing Equations

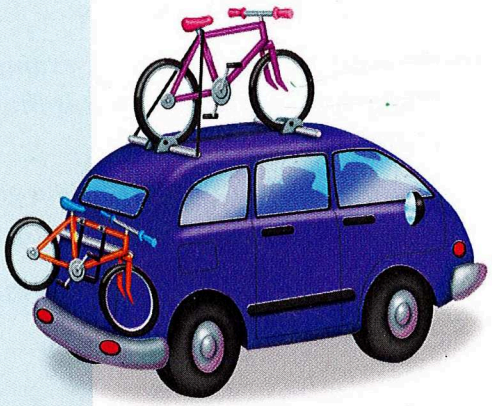
On the last day of the Ocean Bike Tour, the riders will be near Wild World Amusement Park. Liz and Malcolm want to plan a stop there. They consider several variables that affect their costs and the time they can spend at Wild World.

Getting Ready for Problem 3.1

- What variables do you think are involved in planning for the amusement-park trip?
- How are those variables related to each other?

B. Copy and complete the graph for all three speeds below. Use a different color for each speed.





C. Do the following for each of the three average speeds:

1. Look for patterns relating distance and time in the table and graph. Write a rule in words for calculating the distance traveled in any given time.
2. Write an equation for your rule, using letters to represent the variables.
3. Describe how the pattern of change shows up in the table, graph, and equation.

D. For each speed, (50, 55, and 60 mph) tell how far you would travel in the given time. Explain how you can find each answer by using the table, the graph, and the equation.

1. 3 hours
2. $4\frac{1}{2}$ hours
3. $5\frac{1}{4}$ hours

E. For each speed, find how much time it will take the students to reach these cities on their route:

1. Atlantic City, New Jersey, about 320 miles from Norfolk
2. Baltimore, Maryland, about $\frac{3}{4}$ of the way from Norfolk to Atlantic City

ACE **Homework starts on page 55.**

3. Write an equation for the admission price p for a group of n people.
4. Sketch a graph to show the admission price for a group of any size.
5. How does the pattern of change show up in the equation and graph? How is this pattern similar to the pattern in Problem 3.1? How is it different?

B. Admission to Wild World includes a bonus card with 100 points that can be spent on rides. Rides cost 6 points each.

1. Copy and complete the table below to show a customer's bonus card balance after each ride. Pay close attention to the values in the Number of Rides row.

Bonus Card Balance

Number of Rides	0	1	2	3	5	7	10	13	16
Points on Card	100								

2. Describe in words how you can calculate the number of points left after any number of rides.
3. Write an equation showing the relation between the number of rides and the points left on the bonus card. Use letters to represent the variables.
4. Sketch a graph of the data.
5. How does the pattern of change between the variables show up in the equation and graph? How is this pattern similar to the pattern in Question A? How is it different?

C. Liz wonders whether they should rent a golf cart to carry the riders' backpacks at the park. The equation $c = 20 + 5h$ shows the cost c in dollars of renting a cart for h hours:

1. Explain what information the numbers and variables in the equation represent.
2. Use the equation to make a table for the cost of renting a cart for 1, 2, 3, 4, 5, and 6 hours.
3. Make a graph of the data.
4. Describe how the pattern of change between the two variables shows up in the table, graph, and equation.

ACE **Homework starts on page 55.**

Applications

1. The El Paso Middle School girls' basketball team is going from El Paso to San Antonio for the Texas state championship game. The trip will be 560 miles. Their bus travels at an average speed of 60 miles per hour.

a. Suppose the bus travels at an almost steady speed throughout the trip. Make a table and a graph of time and distance data for the bus.

b. Estimate the distance the bus travels in 2 hours, $2\frac{3}{4}$ hours, $3\frac{1}{2}$ hours, and 7.25 hours.

c. How are 2 hours and the distance traveled in 2 hours represented in the table? How are they shown on the graph?

d. How are $2\frac{3}{4}$ hours and the distance traveled in $2\frac{3}{4}$ hours represented in the table? How are they shown on the graph?

e. Describe in words a rule you can use to calculate the distance traveled for any given time on this trip.

f. The bus route passes through Sierra Blanca, which is 90 miles from El Paso. About how long does it take the bus to get to Sierra Blanca?

g. The bus route also passes through Balmorhea, which is $\frac{1}{3}$ of the way from El Paso to San Antonio. About how long does it take the bus to get to Balmorhea?

h. How long does it take the bus to complete its 560-mile trip to San Antonio?

8. Sean is buying a new DVD player and speakers for $315. The store offers him an interest-free payment plan that allows him to pay in monthly installments of $25.

a. How much will Sean still owe after one payment? After two payments? After three payments?

b. Use n to stand for the number of payments and a for the amount still owed. Write an equation for calculating a for any value of n.

c. Use your equation to make a table and a graph showing the relationship between n and a.

d. As n increases by 1, how does a change? How is this change shown in the table? How is it shown on the graph?

e. How many payments will Sean have to make in all? How is this shown in the table? How is this shown on the graph?

For Exercises 9–12, express each rule as an equation. Use single letters to stand for the variables. Identify what each letter represents.

9. The area of a rectangle is its length multiplied by its width.

10. The number of hot dogs needed for the picnic is two for each student.

11. The amount of material needed to make the curtains is 4 square yards per window.

12. Taxi fare is $2.00 plus $1.10 per mile.

13. The sales tax in a state is 8%. Write an equation for the amount of tax t on an item that costs p dollars.

14. An airplane is traveling at 550 miles per hour. Write an equation for the distance d the plane travels in h hours.

15. Potatoes sell for $0.25 per pound at the produce market. Write an equation for the cost c of p pounds of potatoes.

16. A cellular family phone plan costs $49 per month plus $0.05 per minute of long-distance service. Write an equation for the monthly bill b when m minutes of long-distance service are used.

25. the 10th odd number (1 is the first odd number, 3 is the second odd number, and so on.)

26. the area of a triangle with a base of 10 centimeters and a height of 15 centimeters

27. $3^3 \times 5^2 \times 7$

28. The wheels on Kai's bike are 27 inches in diameter. His little sister, Masako, has a bike with wheels that are 20 inches in diameter. Kai and Masako are on a bike ride.

a. How far does Kai go in one complete turn of his wheels?

b. How far does Masako go in one complete turn of her wheels?

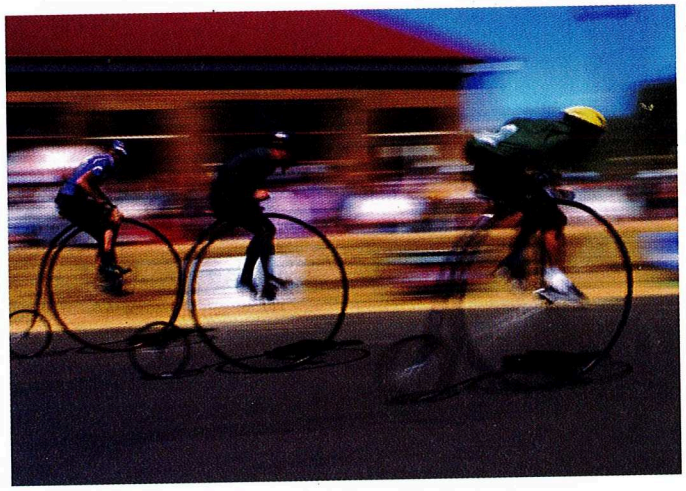
c. How far does Kai go in 500 turns of his wheels?

d. How far does Masako go in 500 turns of her wheels?

e. How many times do Kai's wheels have to turn to cover 100 feet?

f. How many times do Masako's wheels have to turn to cover 100 feet? To cover 1 mile?

29. Bicycles that were popular in the 1890s were called "penny farthing" bicycles. These bikes had front wheels with diameters as great as 5 feet! Suppose the front wheel of these bicycles have a diameter of 5 feet.



a. What is the radius of the front wheel?

b. How far will one bike travel in 100 turns of the front wheel?

c. How many times will the front wheel turn in a 3-mile trip?

d. Compare the number of times the wheels of Masako's bike turn in a 1-mile trip [see part (f) of Exercise 28] with the number of times the front wheel of this penny-farthing bike turns in a 3-mile trip. Why are the numbers related this way?

Extensions

43. a. You can calculate the average speed of a car trip if you know the distance and time traveled. Copy and complete the table below.

Car Trips

Distance (mi)	Time (hr)	Average Speed (mi/h)
145	2	■
110	2	■
165	2.5	■
300	5.25	■
446	6.75	■
528	8	■
862	9.5	■
723	10	■

b. Write a formula for calculating the average speed s for any given distance d and time t.

For Exercises 44–47, solve each problem by estimating and checking.

44. The equation $p = 50 + 10n$ gives the admission price p to Wild World for a group of n people. A club's budget has \$500 set aside for a visit to the park. How many club members can go?

45. The equation $b = 100 - 6r$ gives the number of bonus points b left on a Wild World bonus card after r rides.

a. Rosi has 34 points left. How many rides has she been on?

b. Dwight has 16 points left. How many rides has he been on?

46. The equation $d = 2.5t$ describes the distance in meters d covered by a canoe-racing team in t seconds. How long does it take the team to go 125 meters? How long does it take them to go 400 meters?

47. The equation $d = 400 - 2.5t$ describes the distance in meters d of a canoe-racing team from the finish line t seconds after a race starts. When is the team 175 meters from the finish line? When is it 100 meters from the finish line?